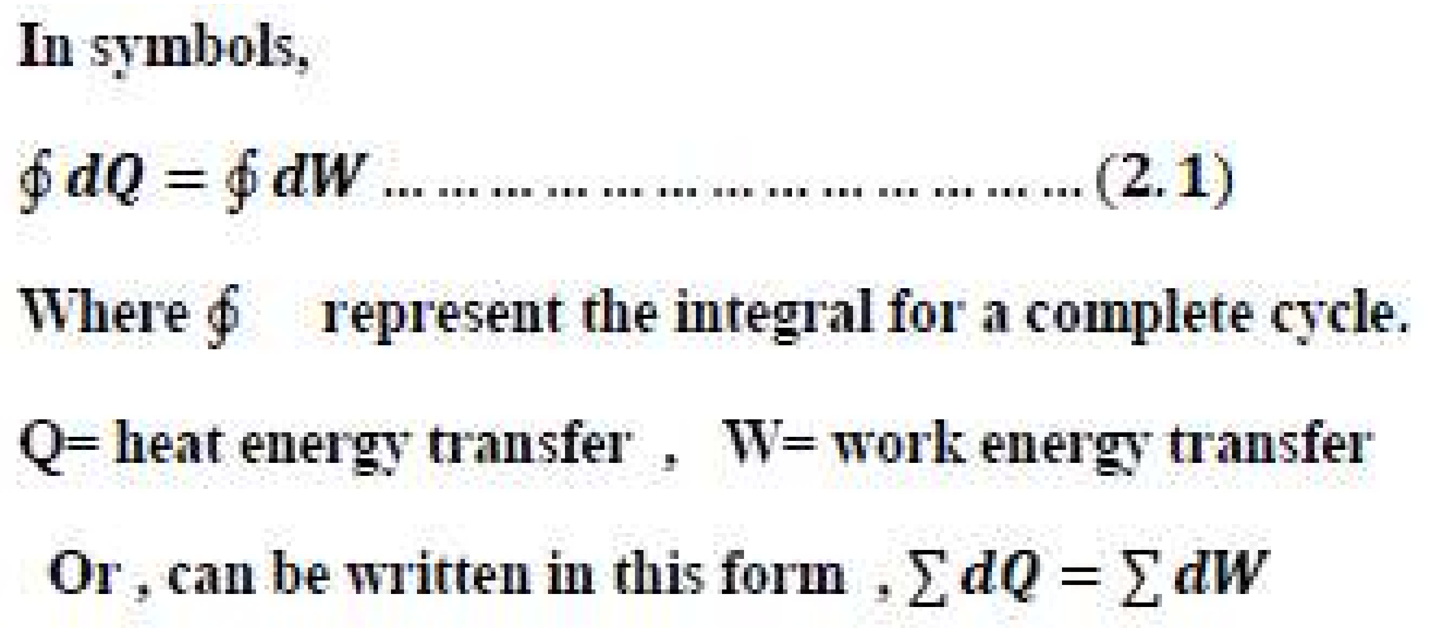
Chapter Two
The First Law of Thermodynamics

2.1 Introduction

When a system undergoes a thermodynamic cycle then the net heat supplied to the system from its surroundings is equal to the net work done by the system on its surroundings.



In symbols,

$$\oint dQ = \oint dW \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2.1)$$

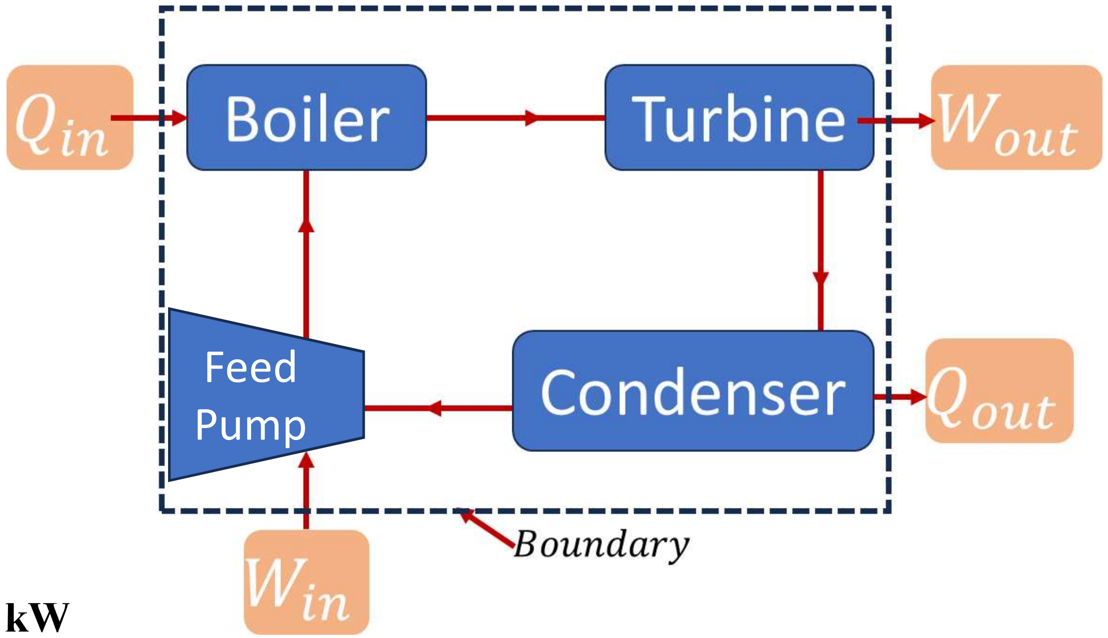
Where $\oint$ represent the integral for a complete cycle.

Q= heat energy transfer , W= work energy transfer

Or , can be written in this form , $\sum dQ = \sum dW$

Example:

In a certain steam plant, the turbine develops 1000kW. The heat supplied to the steam in the boiler is 2800 KJ/kg, the heat rejected by the system to cooling water in the condenser is 2100 KJ/kg and the feed pump work required to pump the condensate back into the boiler is 5 KW. Calculate the steam flow around the cycle in kg/s.



Given :

Work output from the turbine (from the system)=W_{out} = 1000 kW

Heat supplied to the steam in the boiler (system) =Q_{in} = 2800 kJ/kg

Heat rejected by the steam(system) from the condenser =Q_{out}= 2100 kJ/kg

Work input to the pump=W_{in} = 5 kW

From the condenservation of energy

(First law of thermodynamics)equation:

$$\sum dQ = \sum dW \ \dots\dots\dots\dots\dots\dots.(1),$$

$$\sum dQ = Q_{in} - Q_{out} = 2800 - 2100 = 700\,\frac{kJ}{kg}$$

$$\sum dW = W_{out} - W_{in} = 1000 - 5 = 995\ kW,$$

$$700 \times m_{steam} = 995, m_{seam} = \frac{995}{700} = 1.421\ kg/s$$

2.2 Corollaries of the First Law of Thermodynamics:

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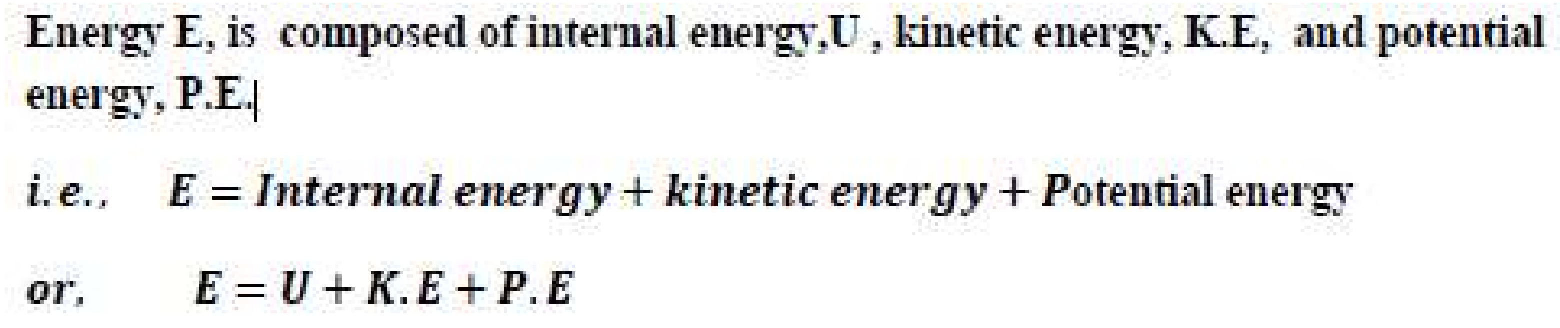
The first law of thermodynamics, in the form of equation (2.1) , has a number of important consequences(or values) which are stated in the form of corollaries(or results):

2.2.1 Corollary 1

There exists a property of a closed system called energy such that a change in its value is equal to the difference between the heat supplied and the work done during any change of state. In other words, there exists a point function such that

$\boldsymbol{\delta Q - \delta W = \Delta E = E_2 - E1}$ $or, \quad Q - W = \Delta E = E_2 - E1$……………..(2.2)



Energy E, is composed of internal energy,U , kinetic energy, K.E, and potential energy, P.E.

$i.e., \quad E = Internal\ energy + kinetic\ energy + \mathrm{Potential\ energy}$

$or, \quad E = U + K.E + P.E$

In a closed system kinetic and potential energy effects generally play a negligible part, equation (2.2) may be simplified to :

$$\boldsymbol{Q - W = \Delta U \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(2.3)}$$

The property ***U*** is called the internal energy of the system. The property ***U*** is the outcome of the first law of thermodynamics. Physically, an increase in ***U*** is associated with a rise in the temperature of the system.

2.2.2 Corollary 2:

Conservation of energy The internal energy of a closed system remains unchanged if the system is isolated from its surroundings.

Proof: By definition of an isolated system, $\boldsymbol{Q = W = 0, \Delta U = U_2 - U_1 = 0}$

$$Q = 0$$

(Isolated system)

$$W = 0$$

$$\Delta U = 0$$

2.2.3 Corollary 3, Perpetual motion machine of the first kind

A perpetual motion machine of the first kind (pmmI) is impossible.

Perpetual motion, is the action of a device that, once set in motion, would continue in motion forever, with no additional energy required to maintain it. Such devices are impossible on grounds stated by the first and second laws of thermodynamics.

$$\boxed{\oint \delta Q = \oint \delta W}$$

Equation (2.1) states, that, if a net amount of heat is not supplied during a cycle, no net amount of work can be obtained, that is

$$\boxed{\oint \delta W = 0\,, \qquad if \quad \oint \delta Q = 0}$$

The existence of a pmmI would therefore violate the first law of thermodynamics.

2.2.4 Energy balance

The basis of the first law of thermodynamics is the law of conservation of energy which, for a system, can be expressed as :

Net energy added to a system=Energy in – Energy out= increase in stored energy. But, for a system undergoing a cycle, there is no change in the state of the system so that, the stored energy is zero. Hence :

Energy in = Energy out

2.3 The Perfect Gas

2.3.1 The Characteristic Equation of State for Ideal Gas:

$$\boxed{\frac{PV}{T} = constant = R \ldots}$$

The equation $\frac{PV}{T} = R$, is called the characteristic equation of a state of a perfect gas. The constant R is called the gas constant. Each perfect gas has a different gas constant.
Units of R are Nm/kg.K or kJ/kg.K
Usually, the characteristic equation is written as:

$$Pv = RT \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 2.4 \quad \text{or } PV = mRT \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 2.5$$

- The characteristic equation in another form, can be derived by using kilogram mole as a unit. The kilogram-mole is defined as a quantity of a gas equivalent to M kg of gas, where M is the molecular weight of the gas (e.g., since the molecular weight of oxygen is 32, then 1 kg mole of oxygen is equivalent to 32 kg of oxygen).

$$m = nM \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots . (2.6)$$

Where m=mass , n= number of mole , M= Molecular mass. Substituting for *m* from eq.(2.6) in eqn.(2.5) gives: $PV = nMRT,$ $\quad or \quad$ $MR = \frac{PV}{nT}$

According to Avogadro's hypothesis, the volume of 1 mole of any gas is the same as the volume of 1 mole of any other gas when the gases are at the same temperature and pressure. Therefore $\frac{V}{n}$ is the same for all gases at the same value of p and T. That is the quantity is a constant for all gases. This constant is called the universal gas constant and is given the symbol, R_o.

$$\therefore MR = R_o = \frac{PV}{nT} \ldots\ldots . . 2.7, \qquad or \qquad PV = n\,R_oT, \qquad since\ \ MR = R_o, Then\ R = \frac{R_o}{M} \ldots\ldots\ldots\ldots . . (2.8)$$

It has been found experimentally that the volume of 1 mole of any perfect gas at 1 bar and 0^oC is approximately 22.71 m^3.

There for from eq.(2.7) $R_o = \frac{PV}{nT} = \frac{1\times10^5\times22.72}{1\times(0^oC+273)} = 8314.3\,\frac{Nm}{molr.K} = 8.3143\,\frac{kJ}{mole.K}$

Example : What is the value of gas constant for oxygen that has a molecular weight of 32?

$$R = \frac{R_0}{M} = \frac{8314}{32} = 259.8\,Nm/kg.K$$

2.3.2 Specific Heats

The specific heat of a solid or liquid is usually defined as the heat required to raise unit mass through a one-degree temperature rise.

- for small quantities, we have

$$dQ = mcdT$$

Where m= mass , c=specific heat, and dT= temperature rise

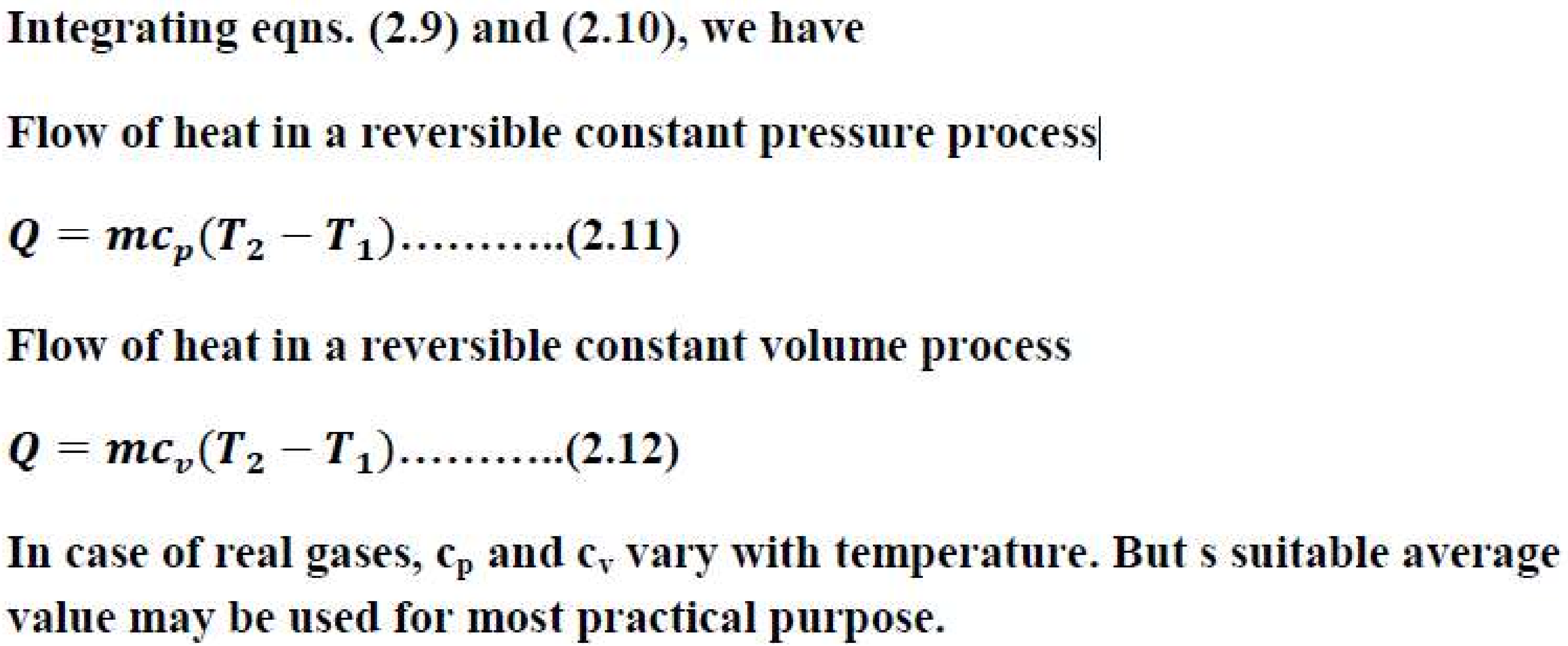
Only two specific heats for gases are defined.

- specific heat at constant volume, cv
-specific heat at constant pressure, cp

We have $dQ = mc_p dT$ for a reversible non-flow process at constant Pressure ……….(2.9)

We have $dQ = mc_p dT$ for a reversible non-flow process at constant Pressure ……….(2.9)

And $dQ = mc_v dT$ for a reversible non-flow process at constant volume ……….(2.10)



Integrating eqns. (2.9) and (2.10), we have

Flow of heat in a reversible constant pressure process

$Q = mc_p(T_2 - T_1)$……….(2.11)

Flow of heat in a reversible constant volume process

$Q = mc_v(T_2 - T_1)$……….(2.12)

In case of real gases, c_p and c_v vary with temperature. But s suitable average value may be used for most practical purpose.

2.3.3 Joule's Law

Joule's law states that:" The internal energy of a perfect gas is a function of the absolute temperature only"

$$i.e \quad u = f(T)$$

To evaluate this function let 1 kg of a perfect gas be heated at constant volume According to the non-flow energy equation,

$$dQ = du + dW$$

$$\therefore Q = du$$

$$dQ = c_v dT$$

$$dQ = du = c_v dT$$

At constant volume for a perfect gas, from eqn.(2.12), for 1 kg

$$Q = mc_v(T_2 - T_1)\text{..........}(2.12)$$

When the volume remains constant the dW = 0

$$\int du = \int c_v dT = u = c_v T + K,$$

$$K\ is\ integral\ constant$$

$$when\ T = 0, u = 0\ (Joule's\ Law)\ \therefore K = 0$$

$$u = c_v T \ldots\ldots\ldots\ldots\ldots.\ldots(2.13)$$

$$For\ mass\ m$$

$$U = m.c_v.T \ldots\ldots..(2.14)$$

Gain in internal energy,

$$U_2 - U_1 = m.c_v\,(T_2 - T_1) \ldots\ldots\ldots\ldots\ldots\ldots (2.15)$$

Equation (2.15) gives the gains of internal energy for a perfect gas between two states for any process, reversible or irreversible.

2.3.4 Relationship Between two Specific Heats

Consider a perfect gas being heated at *constant pressure* from T1 to T2 . According to non-flow equation,

$$Q = (U_2 - U_1) + W$$

$$U_2 - U_1 = m\,c_v(T_2 - T_1)$$

$$Q = m\,c_v(T_2 - T_1) + W$$

In a constant pressure process, the work done by the fluid,

$$W = P(V_2 - V_1) = mR(T_2 - T_1)$$

$$P_1V_1 = mRT_1, \qquad and \quad P_2V_2 = mRT_2$$

$In\ a\ constant\ process\ P_1 = P_2 = P\ in\ this\ case$

$$Q = m\,c_v(T_2 - T_1) + mR(T_2 - T_1)$$

But for constant pressure process,

$$Q = m\,c_p(T_2 - T_1)$$

By equating the two expression, we have

$$m\,(c_v + R)(T_2 - T_1) = m\,c_p(T_2 - T_1)$$

$$\therefore\ (c_v + R) = c_p$$

$$c_p - c_v = R \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(2.16)$$

$divided\ both\ sides\ by\ \ c_v$

$$\frac{c_p}{c_v} - 1 = \frac{R}{c_v}, \qquad if\ \frac{c_p}{c_v} = \gamma(specific\ ratio)$$

$$\therefore\ \gamma - 1 = \frac{R}{c_v}$$

$$,\qquad or \quad c_v = \frac{R}{\gamma - 1} \ldots\ldots\ldots\ldots(1.16a)$$

Similiarly by dividing eq (2.16) both side by c_p we can get

$$c_p - c_v = R \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(2.16)$$

$$c_p = \frac{\gamma R}{\gamma - 1} \ldots\ldots\ldots\ldots\ldots(2.16b)$$

The specific ratio(γ) also varies with temperature, but this variation is very mild. For monatomic gases, its value is essentially constant at γ =1.667. Many diatomic gases, including air, have a specific heat ratio of about γ_{Air} =1.4 at room temperature.

2.3.5 Enthalpy

In thermodynamics is the sum of internal energy(u) and pressure volume product(pv). This sum is called Enthalpy (h)

$$h = u + Pv \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2.17)$$

the total enthalpy of mass, m , of a fluid can be

$$\mathrm{H} = (U + PV), \qquad where\ H = mh$$

For a perfect gas,

$$h = u + pv = c_v T + RT = (c_v + R)T = c_p T$$

$$H = m c_p T,$$

2.4 Application of first law of thermodynamics to non-flow or closed system

1. Réversible constant volume process

(v=constant) In a constant volume process:

(i) the working substance is contained in a rigid vessel

(ii) work=0 , since v=constant

Considering mass of the working substance unity and applying first law of thermodynamics to the process

$$\boldsymbol{Q = (u_2 - u_1) - W \ldots\ldots\ldots\ldots\ldots (2.18)}$$

$$The\ work\ \ W = \int_1^2 Pdv = P(v_2 - v_1) = 0, as\ dv = 0$$

$$\boldsymbol{\therefore Q = (u_2 - u_1) = c_v(T_2 - T_1) \ldots\ldots\ldots\ldots\ldots (2.18a),} \qquad [\boldsymbol{U = mu}]$$

$$\boldsymbol{Q = (U_2 - U_1) = mc_v(T_2 - T_1)}\ldots\ldots\ldots\ldots\ldots\text{(2.18b)}$$

Constant Volume Process

- Constant-volume process---**isochoric process**, volume does not change during the process.

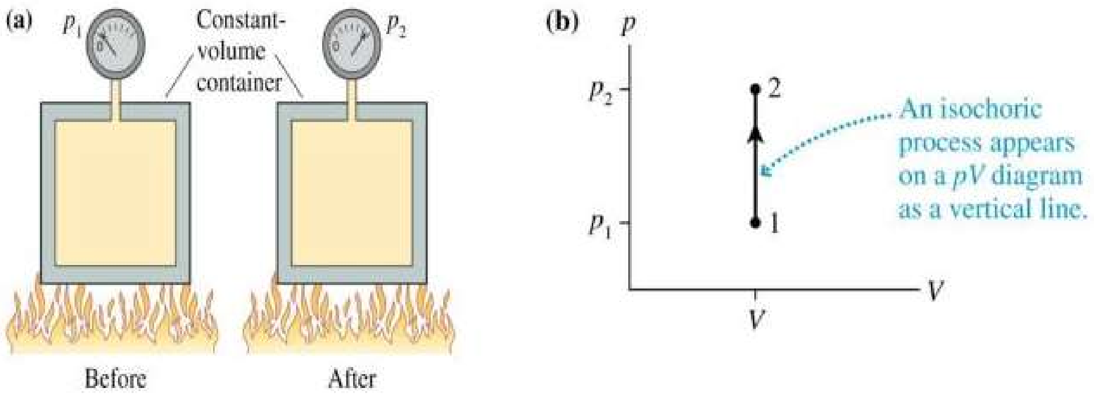


Fig.2.3 : Réversible constant volume process

2. Reversible constant pressure process (p=constant)

Considering unit mass of working substance and Appling first law of thermodynamics to the process,

i.e., Non-Flow Energy Equation(NFEE):

$$\boldsymbol{Q = (u_2 - u_1) + W}$$

The work done

$$\boldsymbol{W = \int_1^2 pdv = p(v_2 - v_1) = pv_2 - pv_1}$$

$$\boldsymbol{Q = (u_2 + p_2v_2) - (u_1 + p_1v_1) = h_2 - h_1}$$

$$[\because\ p_1 = p_2 = p, \qquad h = u + pv]$$

$$\boldsymbol{Q = h_2 - h_1 = c_p(T_2 - T_1)}$$

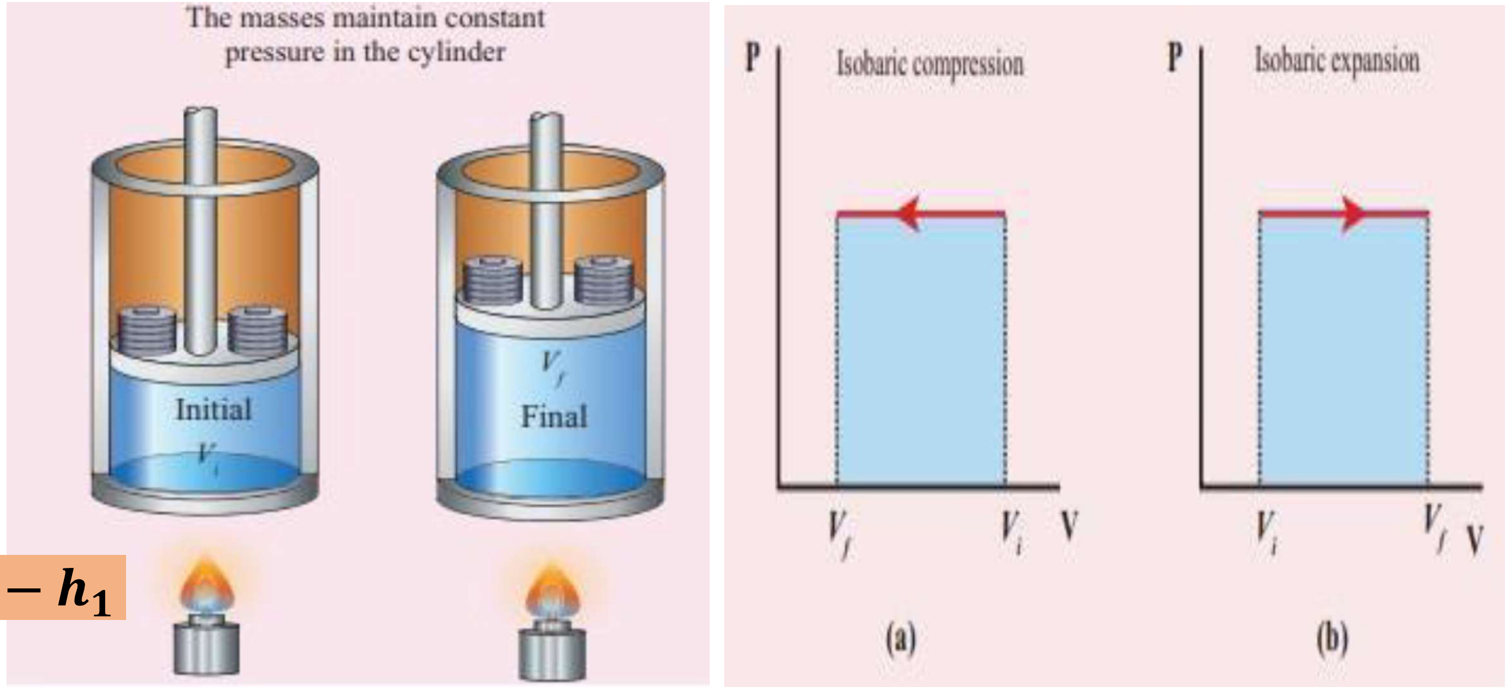


Fig.2.4 Reversible constant pressure process

Where *h*= Enthalpy (specific), and $c_p = \boldsymbol{specific\ heat\ at\ constant\ pressure}$

For mass ,*m*, of working substance

$$\boldsymbol{Q = H_2 - H_1 = mc_p(T_2 - T_1) \ldots\ldots\ldots\ldots\ldots\ldots\ldots (2.19)}$$

3. Reversible Temperature or Isothermal Process (pv=constant, T=constant)

A process at a constant temperature is called an isothermal process

Considering unit mass of working substance and applying first law to the process

$$Q = (u_2 - u_1) + W = c_v(T_2 - T_1) + W = 0 + W$$

$$because\ \ T_1 = T_2$$

$$In\ this\ case\ \ pv = c\ or\ p = \frac{c}{v}$$

$$,\qquad the\ work\ done\ , W$$

$$= \int_{v_1}^{v_2} c\frac{dv}{v} = c[lnv]_{v_1}^{v_2} = cln\frac{v_2}{v_1}$$

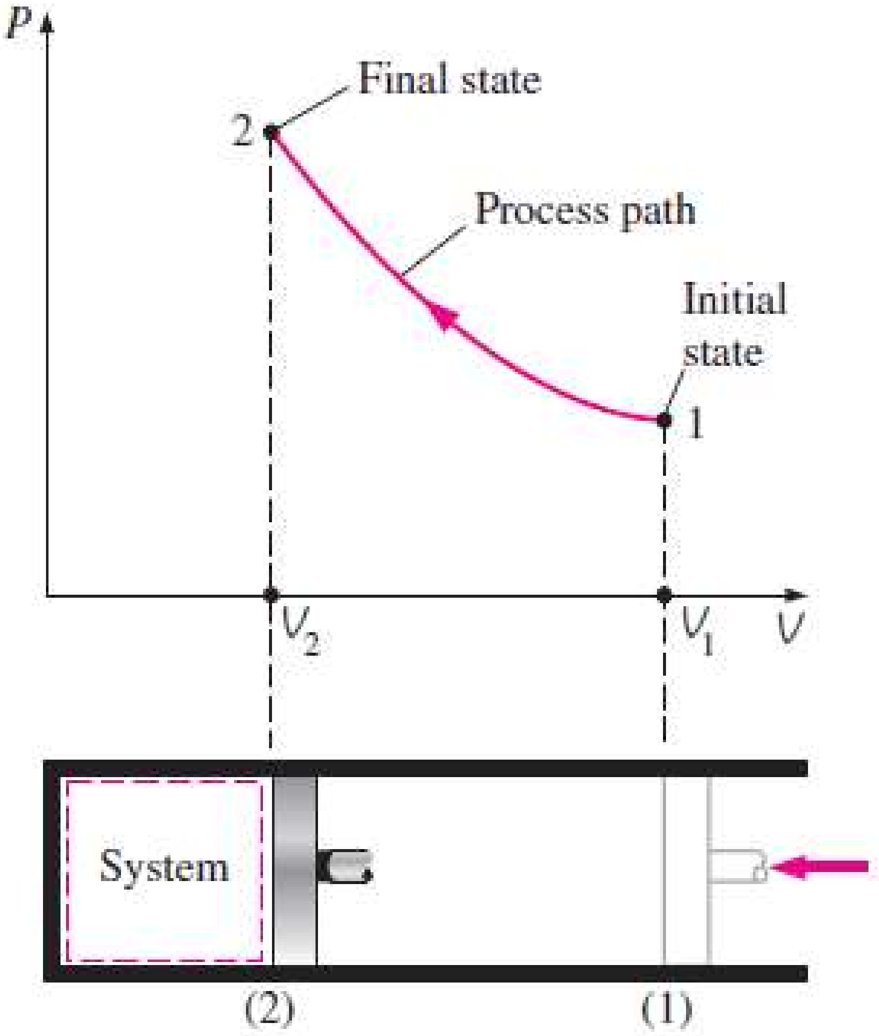


Fig.2.6. Reversible isothermal process

The constant C can either be written as p_1v_1 or as p_2v_2 . since

$$p_1v_1 = p_2v_2 = constanr, C$$

$$i.e, \quad W = p_1v_1 ln\frac{v_2}{v_1} \text{ per unit mass of working substance}$$

$$\therefore \quad Q = W = p_1v_1 ln\frac{v_2}{v_1} \qquad \ldots\ldots\ldots\ldots(2-20)$$

For mass ,m, of working substance

$$\therefore \quad Q = W = p_1V_1 ln\frac{V_2}{V_1} \qquad (2-20a)$$

For isothermal process : $p_1V_1 = p_2V_2$

$$\therefore \quad \frac{V_2}{V_1} = \frac{P_1}{P_2}$$

Or, $\quad Q = W = p_1V_1 ln\frac{p_1}{p_2} \qquad \ldots\ldots\ldots(2-20b)$

4. Reversible Adiabatic Process $pv^{\gamma} = c$

An adiabatic process is one in which no heat is transferred to or from the Fluid (system) during the process (i.e Q=0).

Such a process can be reversible or irreversible. The reversible adiabatic nonflow process will be considered in this section.

Considering unit mass of working substance and applying first law to the process, or Non-Flow Energy Equation (NFEE):

$$Q = (u_2 - u_1) + W, \qquad Since \quad Q = 0, the\ equation\ becomes \qquad 0 = (u_2 - u_1) + W$$

$$\therefore \quad W = (u_1 - u_2) \ldots\ldots\ldots\ldots\ldots..(2.21)$$

To drive the general law for an adiabatic process : $pv^{\gamma} = c$

To obtain a law relating p and v for a reversible adiabatic process let us consider the non-flow energy equation in differential form,

$$dQ = du + dW \qquad for\ a\ reversible\ process\ dW = pdv$$

$$\therefore \ dQ = du + pdv = 0$$

Also for a perfect gas

$$pv = RT \; or \; p = \frac{RT}{v}$$

Sub p in previous eq to be

$$du + \frac{RT}{v} dv = 0, \qquad but \; u = c_v T \qquad or \qquad du = c_v dT$$

$$\therefore \; c_v \, dT + \frac{RT}{v} dv = 0, \quad or \; \therefore \; c_v \frac{dT}{T} + \frac{R}{v} dv = 0 \; (dividing \; all \; term \; to \; T)$$

$$By \; integral \; all \; terms \qquad c_c lnT + Rlnv = Constant$$

$$By \; subsstituting \; \; T = \frac{pv}{R} \; in \; equation \; \; to \; become \qquad c_v ln \frac{pv}{R} + Rlnv = Constant$$

Dividing both sides by c_v $\quad ln \frac{pv}{R} + \frac{R}{c_v} lnv = \frac{constat}{c_v} = Constant$

Again : $c_v = \frac{R}{(\gamma - 1)} \qquad or \qquad \frac{R}{c_v} = \gamma - 1$

Hence substituting

$$ln\frac{pv}{R}+(\gamma-1)lnv=constant$$

$$ln\frac{pv}{R}+\ln v^{(\gamma-1)}=constant$$

$$ln\frac{pv\times v^{(\gamma-1)}}{R}=constant$$

$$ln\frac{pv\times v^{\gamma}\times v^{-1}}{R}=constant$$

$$ln\frac{p\times v^{\gamma}}{R}=constant$$

$$\frac{p\times v^{\gamma}}{R}=e^{constant}=constant$$

Since R = gas constant=constant

$$p\times v^{\gamma}=R\times constant=constant$$

$$\therefore p\times v^{\gamma}=constant$$

Expression for Work W :

A reversible adiabatic process for a perfect gas is shown on a $p-v\ diagram$ in Fig. 2-7 :

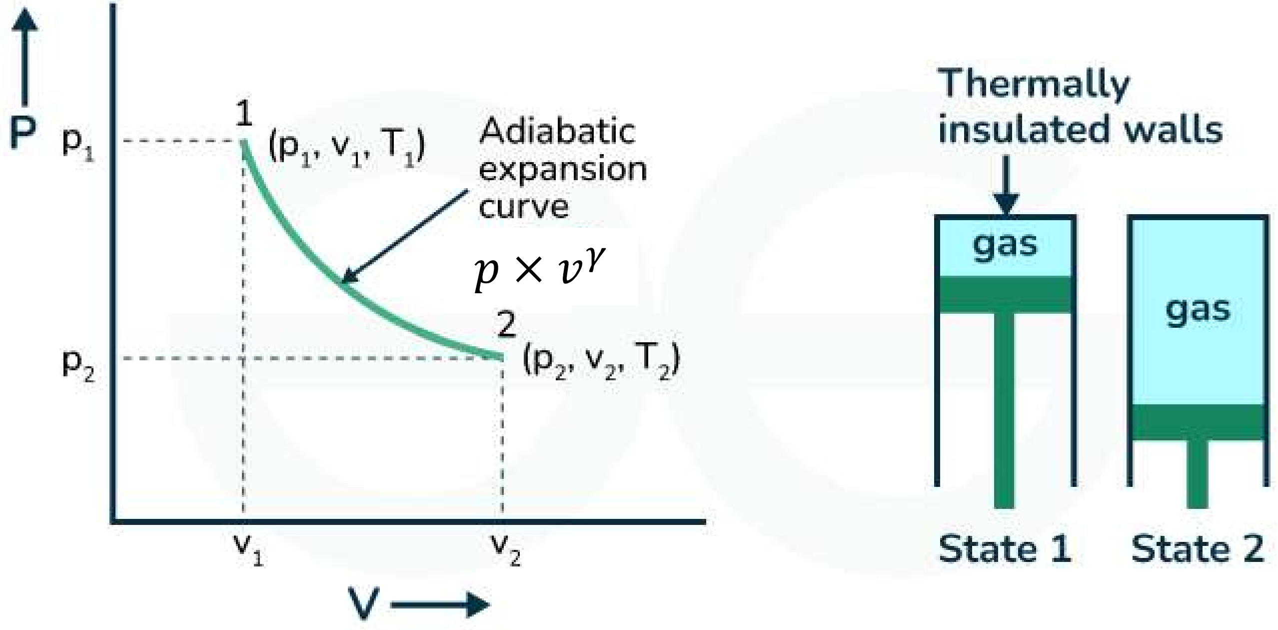


Fig. 2.7 : Reversible adiabatic process

The work of adiabatic process can be evaluated by integration

$$W=\int_{v_1}^{v_2}pdv=\int_{v_1}^{v_2}\frac{C}{v^{\gamma}}dv=C\left|\frac{v^{-\gamma+1}}{-\gamma+1}\right|_{v_1}^{v_2}$$

$$W=\frac{p_1v_1-p_2v_2}{\gamma-1}\dots\dots\dots\dots\dots\dots\dots\dots\dots..(2.22)$$

Relationship between T ,V, and p :

By using equation pv=RT, the relationship between T-v and T-p may be driven as follows.

$$pv = RT \qquad \therefore \quad p = \frac{RT}{v}$$

substituting this value in the equation $pv^{\gamma} = C$ to get $\frac{RT}{v}.v^{\gamma} = C$

$$or \quad \frac{T}{v}.v^{\gamma} = \frac{C}{R} = constant,$$

this can be written as

$$Tv^{\gamma-1} = constant \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 2.23$$

From equation 2.23, $\quad T_1v_1^{\gamma-1} = T_2v_2^{\gamma-1}$

$$\therefore \frac{T_1}{T_2} = \left(\frac{v_2}{v_1}\right)^{\gamma-1} \quad \ldots\ldots\ldots\ldots\ldots\ldots 2.24$$

For an adiabatic process :

$$p_1v_1^{\gamma} = p_2v_2^{\gamma}$$

$$i.e, \quad \frac{p_2}{p_1} = \left(\frac{v_1}{v_2}\right)^{\gamma} \ldots\ldots\ldots(2.25)$$

Also from Eqn.(2.25) :

$$\boxed{\frac{v_1}{v_2} = \left(\frac{p_2}{p_1}\right)^{\frac{1}{\gamma}} \quad \ldots..(2.26)}$$

Substituting Eqn.(2.26) in Eqn. (2.24):

$$\boxed{\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} \quad \ldots\ldots\ldots(2.27)}$$

From eq(2.24) and (2.27) , it can be written that

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{\gamma-1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} \quad \ldots\ldots\ldots(2.28)$$

From eqn. (2.21) , the work done in an adiabatic process per kg of gas is given by

$$W = (u_1 - u_2)$$

$$W = c_v(T_1 - T_2)$$

Also , we know that

$$c_v = \frac{R}{\gamma - 1}$$

Hence, substituting, we get

$$W = \frac{R(T_1 - T_2)}{\gamma - 1}$$

Using equation $pv = RT$

$$W = \frac{(p_1v_1 - p_2v_2)}{\gamma - 1}$$

This is the same expression obtained before as eqn. (2.22)

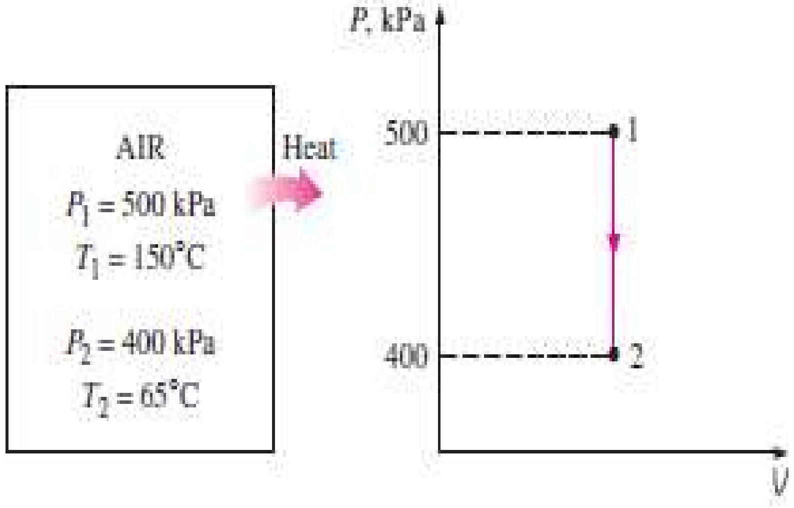
$$\boldsymbol{W = \frac{p_1v_1 - p_2v_2}{\gamma - 1} \dots \dots \dots 2.22)}$$

EXAMPLE 1: EXAMPLE 4–1 Boundary Work for a Constant-Volume Process

A rigid tank contains air at 500 kPa and 150°C. As a result of heat transfer to the surroundings, the temperature and pressure inside the tank drop to 65°C and 400 kPa, respectively. ***Determine the boundary work done during this process*** that are shown in the following figure. (cv=0.718 kJ/kg.K)

Solution 1:

$$W = \int_{v_1}^{v_1} P\,dv = 0$$



Discussion This is expected since a rigid tank has a constant volume and *dV* =0 in this equation. Therefore, ***there is no boundary work done during this process.***

$$Heat\ transfer = the\ change\ of\ internal\ energy$$

$$\therefore\ \ for\ the\ constant\ volume\ process\ the\ \ \ \ Q = (u_2 - u_1) = m\,c_v(T_2 - T_1)$$

$$\therefore\ \ Q = (u_2 - u_1) = 0.718\,(65 - 150) = -61\ kJ/kg$$

$$Q = (u_2 - u_1) = \frac{kJ}{kg.K} \times K\ = -61\ kJ/kg$$

EXAMPLE 2: A frictionless piston–cylinder device contains 4.5 kg of steam at 414 kPa and 433 K. Heat is now transferred to the steam until the temperature reaches 477 K. If the piston is not attached to a shaft and its mass is constant, ***determine the work done by the steam during this process*** that are shown in Fig. Also determine the heat add (Q).(Cp=1.8723 kJ/kg.K Cv=1.4108 kJ/kg.K)

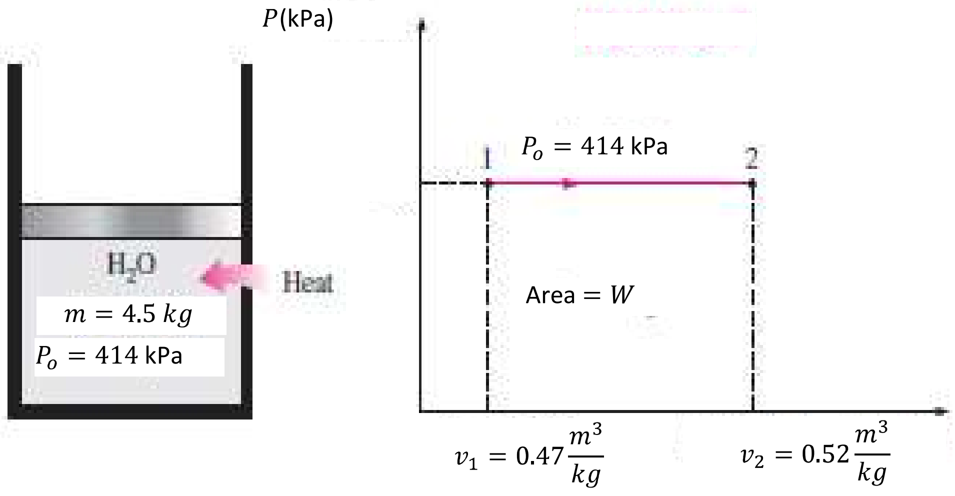
Solution 2:

$$W = \int_{v_1}^{v_2} P\, dv = P \int_{v_1}^{v_2} dv = P(v_2 - v_1)$$

or

$$W = m * P(v_2 - v_1)$$

Where, $V = v * m$



$$W = 4.5 \times 414 \times 1000 \times (0.52 - 0.467) = 98739\ J = 99\ kJ$$

$$\boldsymbol{Q = (u_2 - u_1) + W}$$

$$\boldsymbol{Q = H_2 - H_1 = mc_p(T_2 - T_1) = 4.5 \times 1.8723 \times (477 - 433) = 370\ kJ}$$

$$\boldsymbol{In\ constant\ pressure\ process\ there\ are\ heat\ transfer, change\ in\ internal\ energy, and\ work.}$$

EXAMPLE 3: A piston-cylinder device initially contains 0.4 m^3 of air at 100 kPa and 80°C. The air is now compressed to 0.1 m^3 in such a way that the temperature inside the cylinder remains constant. ***Determine the work done during this process***.

Solution 3:

The temperature remains constant so the process is Isothermal compression

$$PV = mRT = C \quad or \quad P = \frac{C}{V}$$

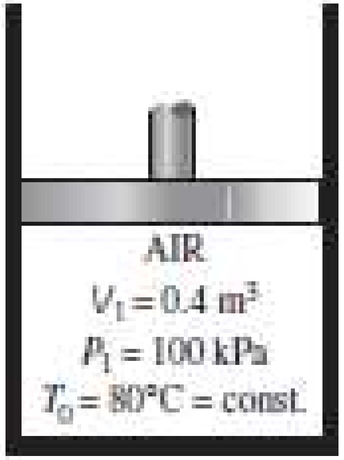
$$W = \int_{V_1}^{V_2} PdV = \int_{V_1}^{V_2} \frac{C}{V} dV = C\, ln \frac{V_2}{V_1} = P_1V_1 ln \frac{V_2}{V_1}$$

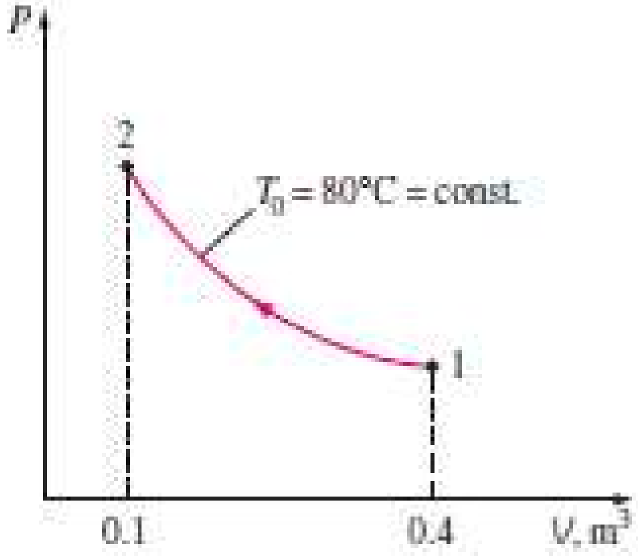
In case we do not have all properties of Air some replacements can be done.

$$P_1V_1 \text{ can be replaced by} P_2V_2 \text{ or by } mRT. Also, \frac{V_2}{V_1} = \frac{P_1}{P_2}$$

$$W = P_1V_1 ln \frac{V_2}{V_1} = 100 \times 0.4 \times \left(ln \frac{0.1}{0.4} \right) = -55.5\ kJ$$

$$W = P_1V_1 ln \frac{V_2}{V_1} = \frac{kN}{m^2} \times m^3 \times (-) = kN.m = kJ$$





Example4: A sample of ideal gas is initially at temperature $T_1 = 400$ K, pressure $p_1 = 5$ bar and occupies a volume $V_1 = 0.6$ m^3. The gas expands adiabatically to a state 2. The processes is reversible.

Givens: R = 8.31 J/mol K; CV = (3/2)R, $V_2 = 3\ m^3$

1.Draw the thermodynamic process in a PV diagram .

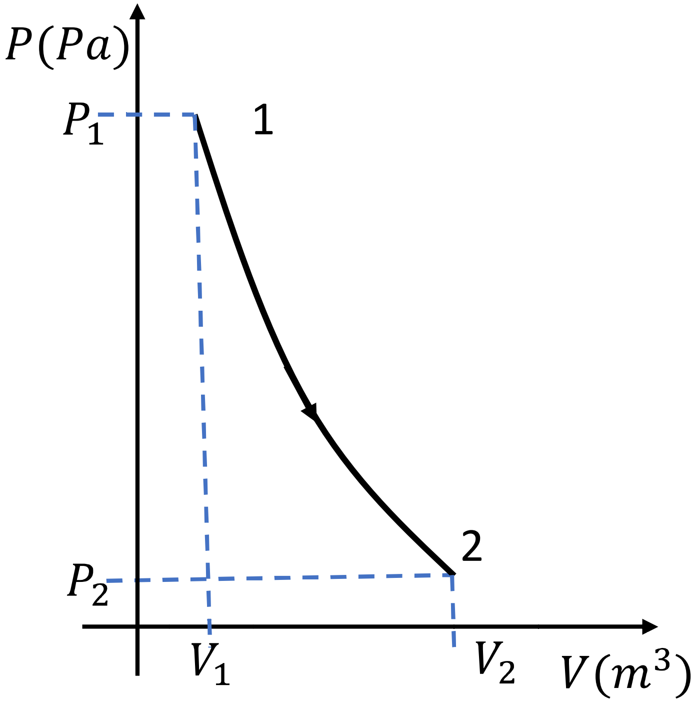
2.Calculate the pressure in state 2 using the adiabatic process equation (between states 1 and 2). What is the temperature in this state?

3.Calculate the work done, the internal energy change and the heat exchanged by the gas.

Solution:

$$\boldsymbol{P_1 V_1^\gamma = P_2 V_2^\gamma}$$

$$P_2 = P_1 \left(\frac{V_1}{V_2}\right)^\gamma = 5 \times 10^5 \times \left(\frac{0.6}{3}\right)^{1.67} = 34016\ Pa$$



We can use the equation of an ideal gas applied to state 2 to calculate the temperature.
But first, we need to determine the number of moles in the sample of ideal gas.

By applying the equation of state of an ideal gas to state 1:

$$P_1 V_1 = n\,R\,T_1$$

$$\boldsymbol{n = \frac{P_1 V_1}{R\,T_1} = \frac{5\times10^5\times0.6}{8.31\times400} = 90}$$

By applying the same equation to state 2:

$$T_2 = \frac{P_2 V_2}{nR} = \frac{34016\times3}{90\times8.31} = 136\ K$$

$$In\ adaibatic\ process\ Q = 0\ , \therefore \mathbf{0} = (\boldsymbol{u_2} - \boldsymbol{u_1}) + \boldsymbol{W},$$

$$\boldsymbol{W = -(u_2 - u_1) = (u_1 - u_2)}$$

$$W_{1-2} = \Delta U = n\,c_v(T_1 - T_2) = 90\times\frac{3}{2}\times8.31\times(400-136) = 296168 = 296kJ$$

5. Polytropic Reversible Process ($pv^n = constant$).

The general law of polytropic (General) process is $pv^n = constant$ where , n is a polytropic index that its value is constant.

We know that for any reversible process, $W = \int pdv$

For a process in $pv^n = constant, we\ have \quad p = \frac{C}{v^n}, where\ C\ is\ a\ constant$

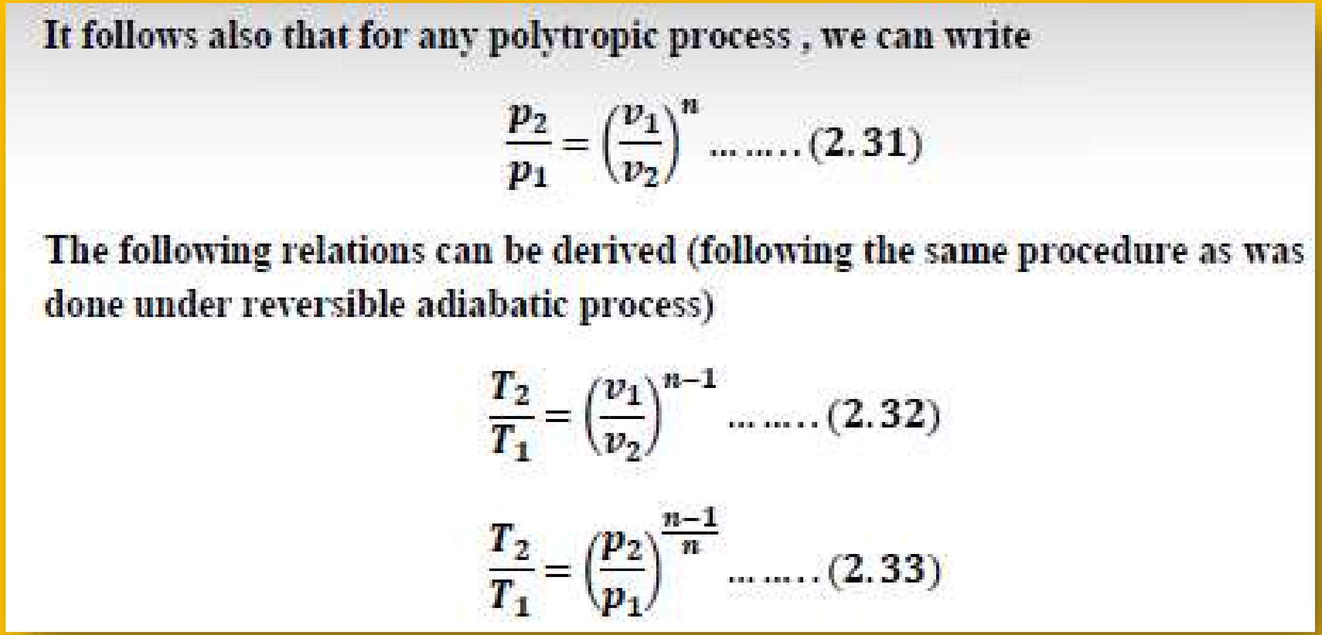
$$\therefore W = C\int_{v_1}^{v_2} \frac{dv}{v^n} = C\left|\frac{v^{n+1}}{-n+1}\right|_{v_1}^{v_2} = C\left(\frac{v_2^{-n+1} - v_1^{-n+1}}{-n+1}\right) = C\left(\frac{v_1^{-n+1} - v_2^{-n+1}}{n-1}\right)$$

Since the constant C can be written as $p_1v_1^n \quad or p_2v_2^n, substituting\ for\ C$

$$W = \left(\frac{p_1v_1^nv_1^{-n+1} - p_2v_2^nv_2^{-n+1}}{n-1}\right)$$

$$, \quad \therefore W = \left(\frac{p_1v_1 - p_2v_2}{n-1}\right) ....2.29)$$

$$or \quad W = \left(\frac{R(T_1 - T_2)}{n-1}\right) ....(2.30)$$



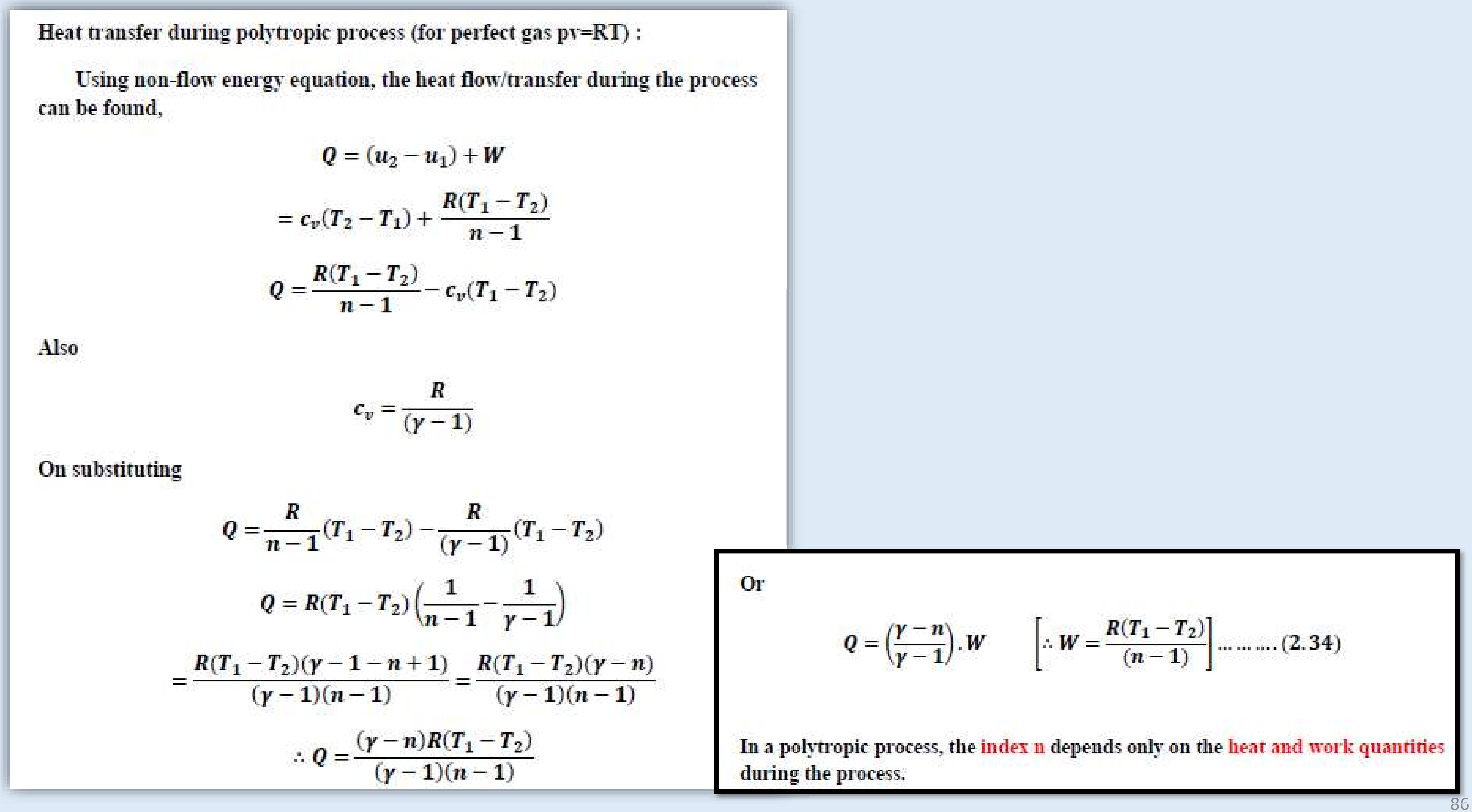
It follows also that for any polytropic process , we can write

$$\frac{p_2}{p_1} = \left(\frac{v_1}{v_2}\right)^n ....(2.31)$$

The following relations can be derived (following the same procedure as was done under reversible adiabatic process)

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{n-1} ....(2.32)$$

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} ....(2.33)$$



Heat transfer during polytropic process (for perfect gas pv=RT) :

Using non-flow energy equation, the heat flow/transfer during the process can be found,

$$Q = (u_2 - u_1) + W$$

$$= c_v(T_2 - T_1) + \frac{R(T_1 - T_2)}{n-1}$$

$$Q = \frac{R(T_1 - T_2)}{n-1} - c_v(T_1 - T_2)$$

Also

$$c_v = \frac{R}{(\gamma - 1)}$$

On substituting

$$Q = \frac{R}{n-1}(T_1 - T_2) - \frac{R}{(\gamma-1)}(T_1 - T_2)$$

$$Q = R(T_1 - T_2)\left(\frac{1}{n-1} - \frac{1}{\gamma - 1}\right)$$

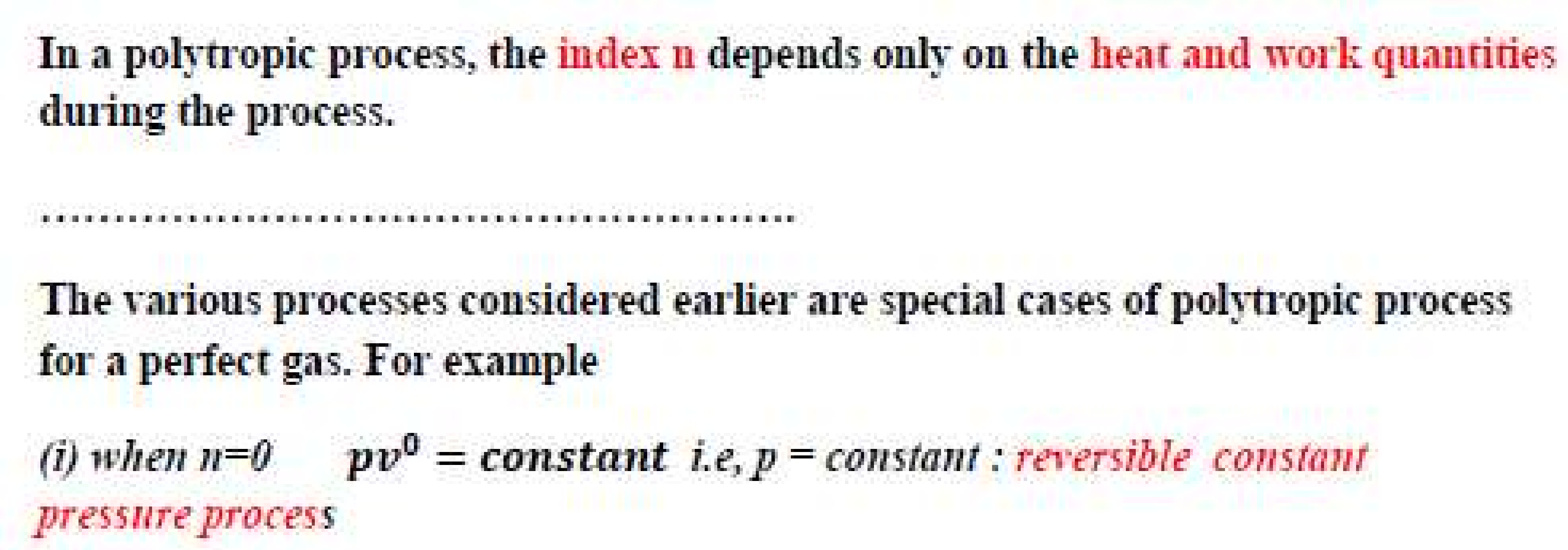
$$= \frac{R(T_1 - T_2)(\gamma - 1 - n + 1)}{(\gamma - 1)(n-1)} = \frac{R(T_1 - T_2)(\gamma - n)}{(\gamma - 1)(n-1)}$$

$$\therefore Q = \frac{(\gamma - n)R(T_1 - T_2)}{(\gamma - 1)(n-1)}$$

Or

$$Q = \left(\frac{\gamma - n}{\gamma - 1}\right).W \qquad \left[\therefore W = \frac{R(T_1 - T_2)}{(n-1)}\right] \ldots\ldots\ldots (2.34)$$

In a polytropic process, the index n depends only on the heat and work quantities during the process.

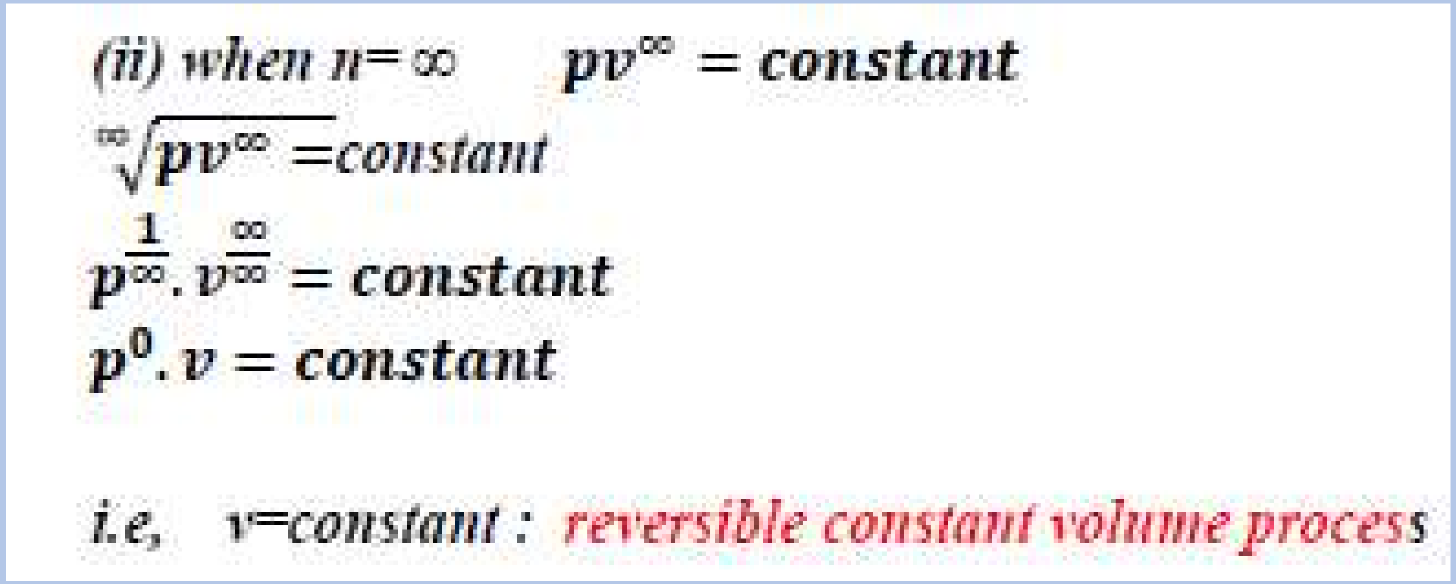


In a polytropic process, the index n depends only on the heat and work quantities during the process.

……………………………………………………

The various processes considered earlier are special cases of polytropic process for a perfect gas. For example

(i) when $n=0$ $\quad pv^0 = constant$ *i.e,* $p = constant$ *: reversible constant pressure process*



(ii) when $n=\infty$ $\quad pv^\infty = constant$

$$\sqrt[\infty]{pv^\infty} = constant$$

$$p^{\frac{1}{\infty}}.v^{\frac{\infty}{\infty}} = constant$$

$$p^0.v = constant$$

i.e, $v = constant$ *: reversible constant volume process*

(iii) when n=1   pv=constant i.e T=constant , since
$pv/T = constant\ for\ gas$: *reversible isothermal process*

(iv) when $n = \gamma$,
$pv^{\gamma} = constant, i, e,$ ***reversible adiabatic process***

(v) when n=n , $pv^{n} = constant,$ ***reversible polytropic process***

This is illustrated on a p-v diagram in Fig. 2.8.

(i) state 1 to state A is *constant pressure* cooling (n=0)

(ii) state 1 to state B is *isothermal compression* (n=1)

(iii) state 1 to state C is *reversible adiabatic compression* $(n = \gamma)$

(iv) state 1 to state D is *constant volume heating* $(n = \infty)$

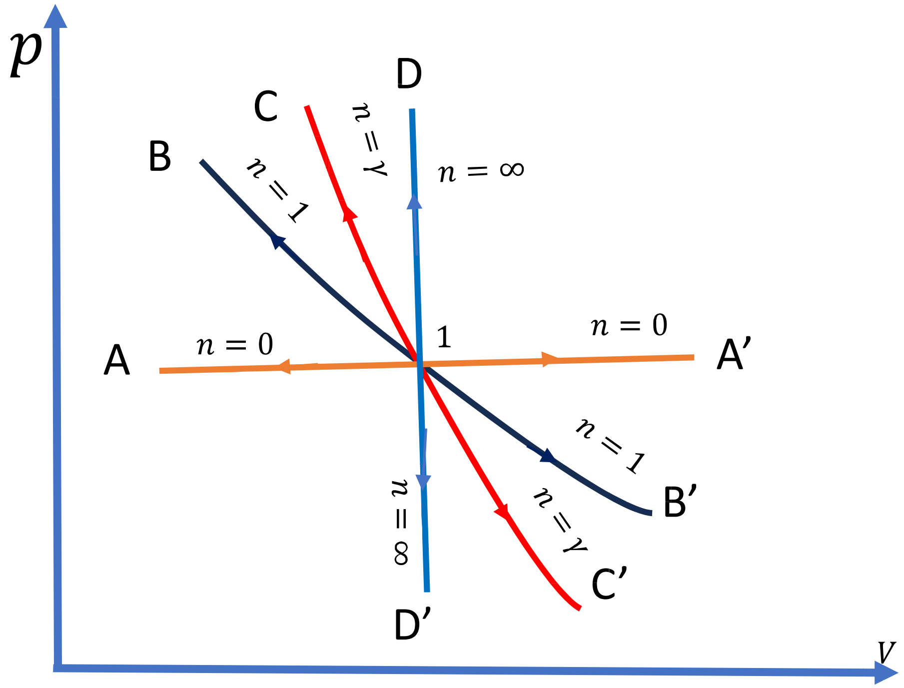
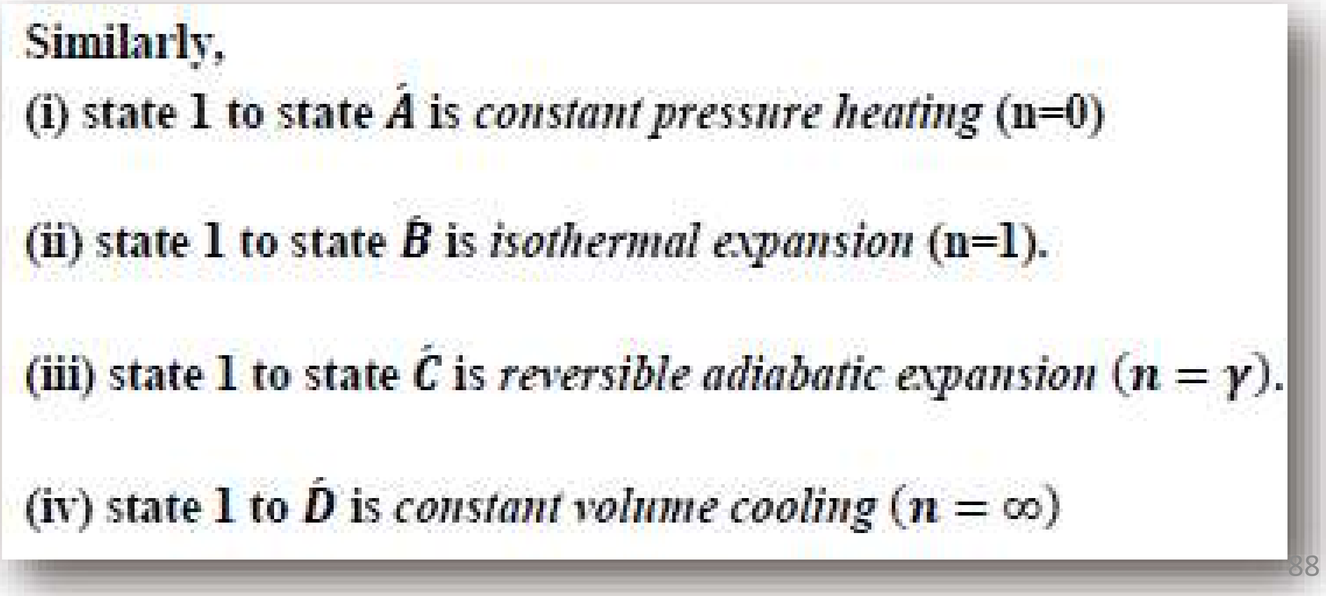


Fig. 2.8.


Similarly,

(i) state 1 to state Á is *constant pressure heating* (n=0)

(ii) state 1 to state B́ is *isothermal expansion* (n=1).

(iii) state 1 to state Ć is *reversible adiabatic expansion* $(n = \gamma)$.

(iv) state 1 to D́ is *constant volume cooling* $(n = \infty)$