2-3 Free Expansion

Consider two vessels 1 and 2 interconnected by a short pipe with a valve A, and perfectly thermally insulated (Fig.2.9). initially let the vessel 1 be filled with a fluid at a certain pressure, and let 2 be completely evacuated. When the valve A is opened the fluid in 1 will expanded rapidly to fill both vessels 1 and 2 . the pressure finally will be lower than the initial pressure in vessel 1 This is known as *Free or unresisted expansion*. The process is highly irreversible.

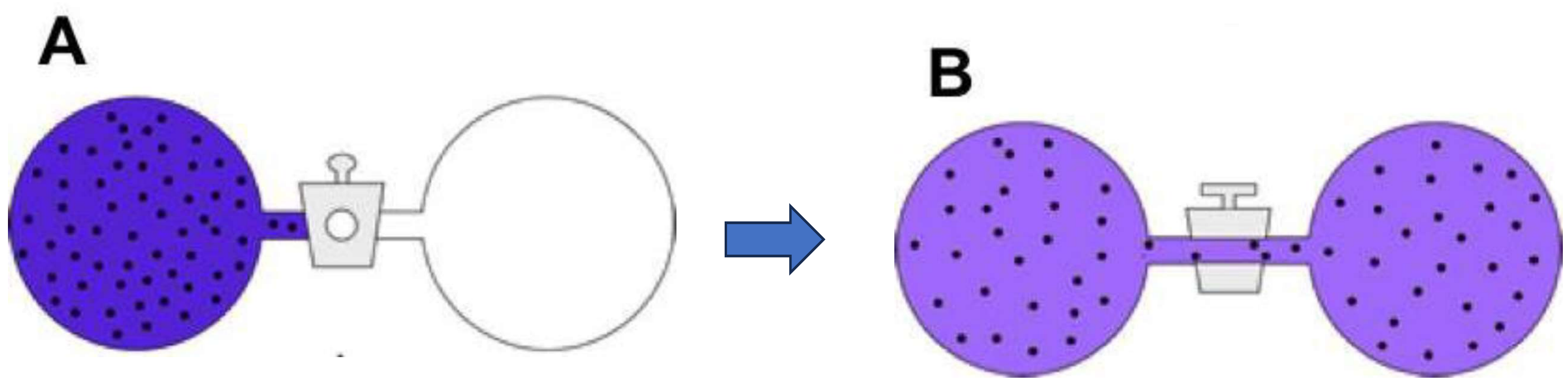


Fig. 2.9 : Free Expansion

Now applying the first law of thermodynamics (or Non-Flow Energy Equation, NFEE) between the initial and final states,

$$\boldsymbol{Q = (u_2 - u_1) + W}$$

In this process, no work is done on or by the fluid, since the boundary of the system does not move. No heat flows to or from the fluid since the system is well insulated. The process is, therefore, *adiabatic* but *irreversible.*

$$\boldsymbol{i.e\ \ (u_2 - u_1) = 0, \qquad or\ u_2 = u_1}$$

In a free expansion, therefore,

The internal energy initially= the initial energy finally

For perfect gas

$$\boldsymbol{u = c_v T}$$

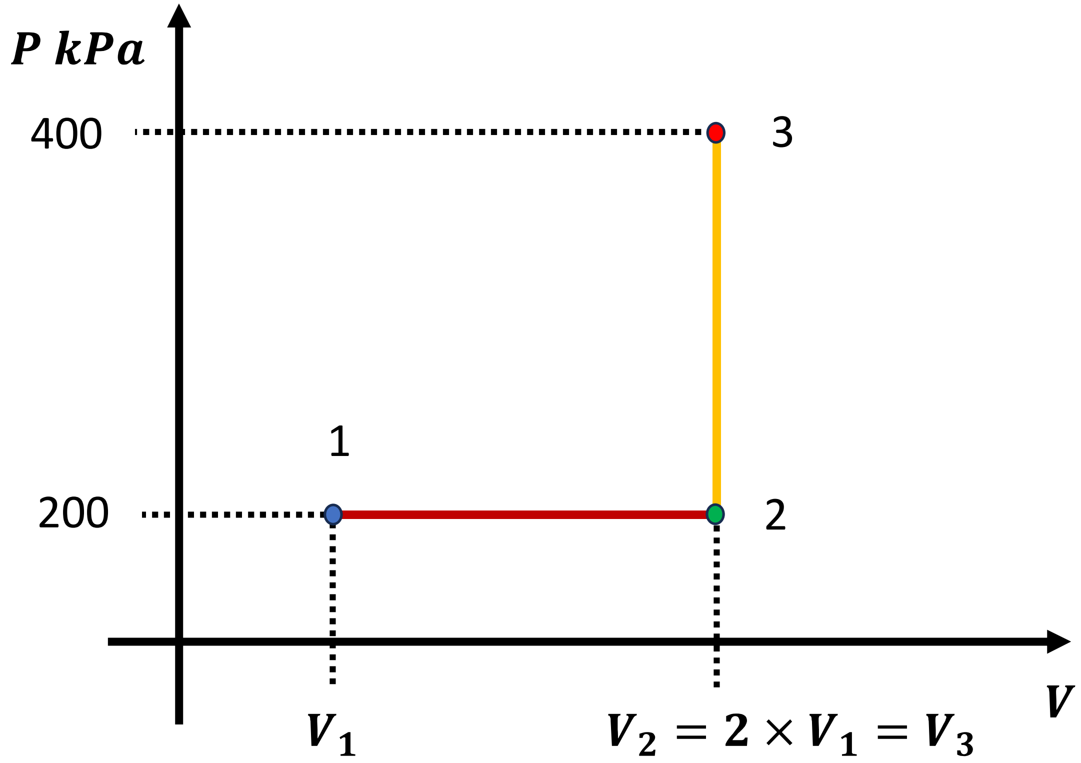
$$\boldsymbol{\therefore For\ free\ expansion\ of\ a\ perfecr\ gas, c_v T_1 = c_v T_2}$$

$$\boldsymbol{i.e. T_1 = T_2}$$

Table 2.1. Summary of Processes for Perfect Gas (Unit mass)

Process	Index n	Heat added	$\int_1^2 pdv$	p, v, T relation	Specific heat, c
Constant pressure	n=0	$c_p(T_2 - T_1)$	$p(v_2 - v_1)$	$\frac{T_2}{T_1} = \frac{v_2}{v_1}$	c_p
Constant volume	$n = \infty$	$c_v(T_2 - T_1)$	0	$\frac{T_1}{T_2} = \frac{p_1}{p_2}$	c_v
Constant temperature	$n = 1$	$p_1 v_1 ln\frac{v_2}{v_1}$	$p_1 v_1 ln\frac{v_2}{v_1}$	$p_1 v_1 = p_2 v_2$	∞
Reversible adiabatic	$n = \gamma$	0	$\frac{p_1 v_1 - p_2 v_2}{\gamma - 1}$	$p_1 v_1^{\gamma} = p_2 v_2^{\gamma}$ $\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{\gamma-1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$	0
Polytropic process	$n = n$	$= \frac{\gamma - n}{\gamma - 1} \times W_{polytropic}$	$\frac{p_1 v_1 - p_2 v_2}{n - 1}$	$p_1 v_1^{n} = p_2 v_2^{n}$ $\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{n-1} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}$	$c_n = c_v\left(\frac{\gamma - n}{1 - n}\right)$

Example5: **2 kg of an ideal gas with constant specific heats begins a process at 200 kPa and temperature (60 oC). The gas is first expanded at constant pressure until its volume doubles. Then it is heated at constant volume until its pressure doubles. Calculate the work done by the gas during the entire process and the final temperature.** ***Take, R***= $0.6\,\frac{kJ}{kg.K}$ *and* $c_v = 2\,\frac{kJ}{kg.K}$



By applying the gas equation

$p_1V_1 = mRT_1$ $\quad$ $T_1 = 60 + 273 = 333\ K$

$200 \times V_1 = 2 \times 0.6 \times (333)$

$V_1 = 1.998\ m^3$

$V_2 = 2V_1 = 2 \times 1.998 = 3.996 m^3$

$w_{12} = p_1(V_2 - V_1) = 200(3.996 - 1.998 = 399.6\ kJ$

$w_{23} = 0\ \ since\ for\ constant\ volume\ process\ dv = 0$

$w_{net} = w_{12} + w_{23} = 399.6 + 0 = 399.6\ kJ$

$p_3V_3 = mRT_3$

$400 \times 3.996 = 2 \times 0.6T_3$

$T_3 = 1332K$

If you asked to calculate the heat transfer Q

For gas : $p_2 V_2 = mRT_2$

$$200 \times 3.996 = 2 \times 0.6 \times T_2$$

$$T_2 = 666\ K$$

For constant pressure process 12, heat transfer during this process is :

$$Q_{12} = mc_p(T_2 - T_1) = 2 \times 2(666 - 333) = 1332 KJ$$

$$R = c_p - c_v$$

$$c_v = c_p - R = 2 - 0.6 = 1.4\ KJ/kg.K$$

For constant volume process 23, heat transfer during this process is :

Since W=0 , for constant volume process, hence

$$Q_{23} = \Delta U = U_3 - U_2 = mc_v(T_3 - T_2) = 2 \times 1.4(1332 - 666) = 1864.8\ KJ$$

$$Q_{net} = Q_{12} + Q_{23} = 1332 + 1864.8 = 3196.8 KJ$$

Example 6: ***In an internal combustion engine, during the compression stroke the heat rejected to the cooling water is*** $50\,\frac{kJ}{kg}$ ***and the work input is*** $100\,\frac{kJ}{kg}$ ***. Calculate the change in internal energy of the working fluid stating whether it is a gain (increase) or loss (Decrease).***

Solution:

Heat rejected to the cooling water,

$$Q = -50\,kJ/kg$$

(-ve sign since heat is rejected)

Work input, $\quad$ **W= -100** kJ/kg

(-ve sign since work is supplied to the system)

Using the relation,

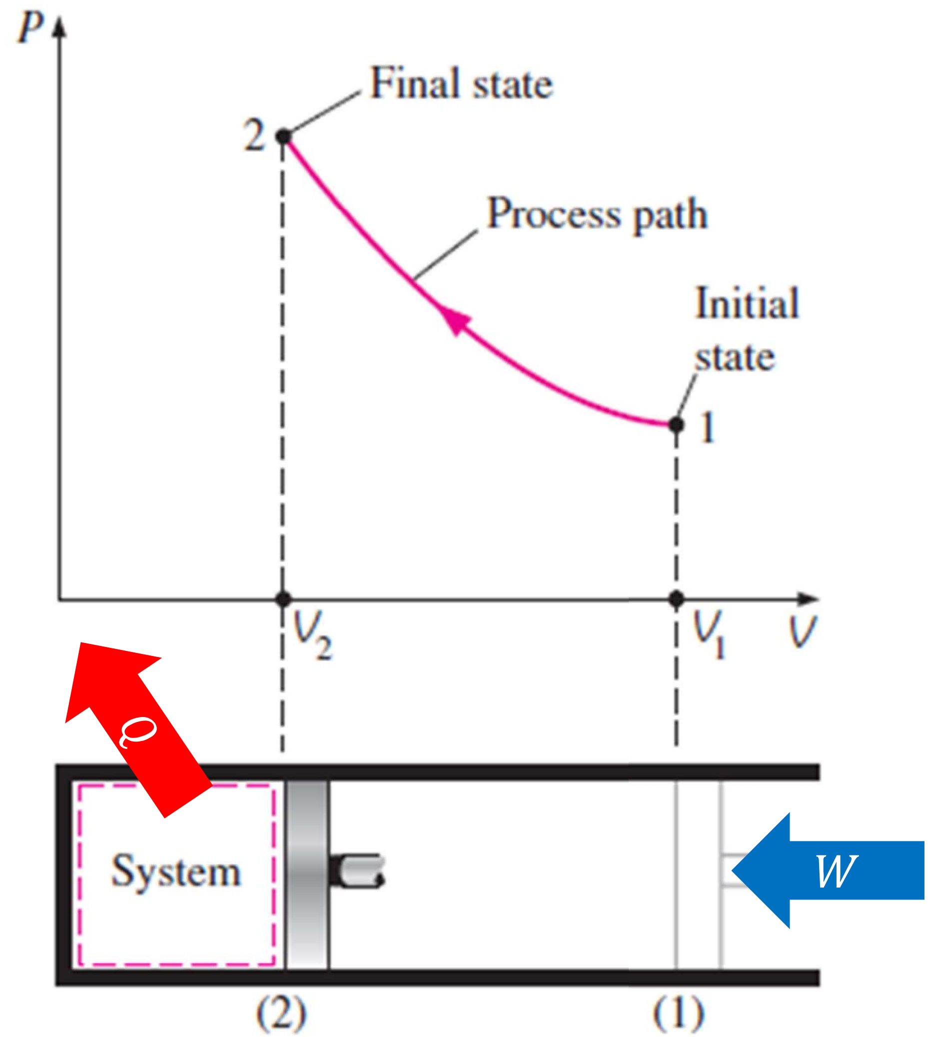
$$Q = (u_2 - u_1) + W$$

$$-50 = (u_2 - u_1) - 100$$

Or

$$u_2 - u_1 = -50 + 100 = 50\;kJ/kg$$

Hence , gain in internal energy = 50 kJ/kg



Example 7: ***0.3 kg of nitrogen gas at 100 kPa and 40^oC is contained in a cylinder. The piston is moved compressing nitrogen until the pressure becomes 1 MPa and temperature becomes 160^oC. The work done during the process is 30 kJ. Calculate the heat transferred from the nitrogen (i.e system) to the surrounding***

Given cv for nitrogen = 0.75 $\frac{kJ}{kg.K}$

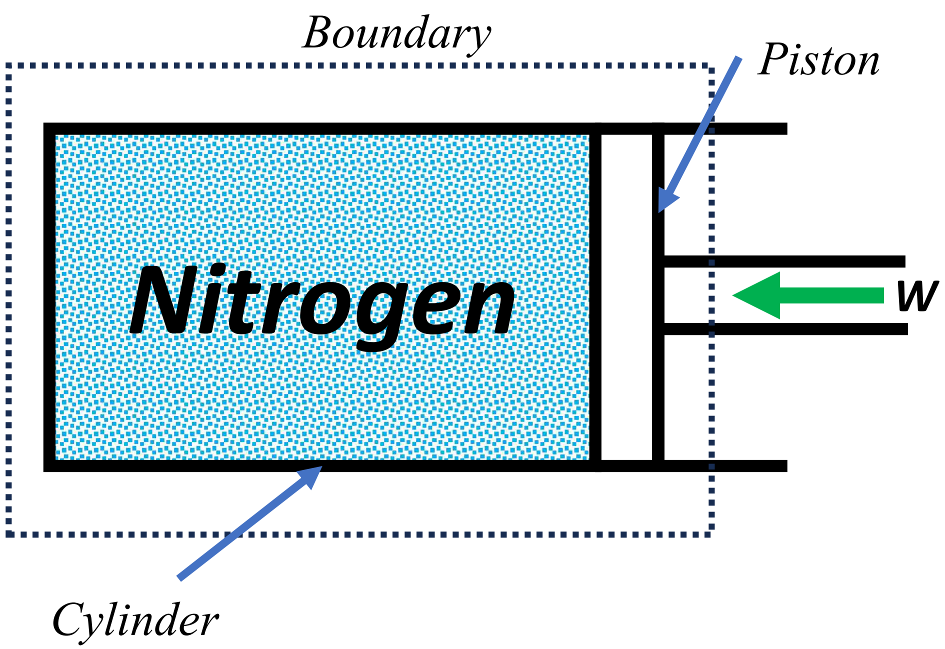
Solution:

Mass(m)=0.3 kg of nitrogen gas

P_1 = 100 kPa and $T_1 = 40^oC$ (40+273=313 K)

P_2 =1 MPa and $T_2 = 160^oC\ (160 + 273 = 433\ K)$.

The work done during the process W= -30 kJ.



According to first law of thermodynamics.

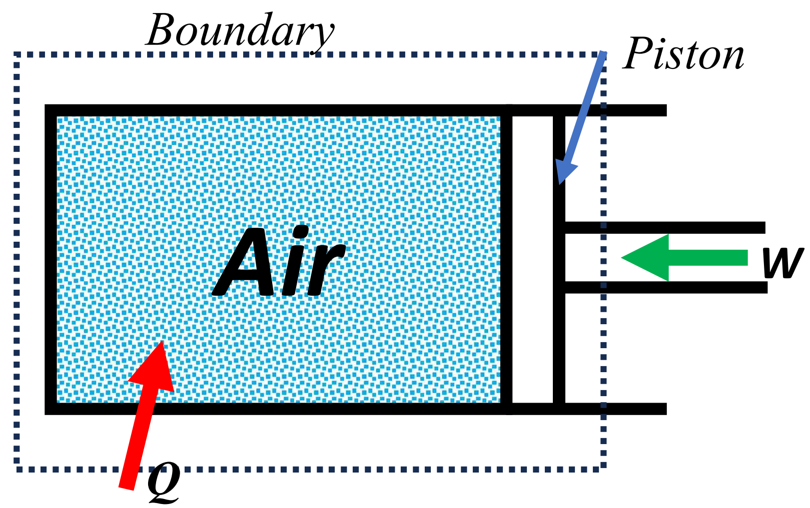
$$Q = \Delta U + W = (U_2 - U_1) + W$$
$$= mc_v(T_2 - T_1) + W$$

$$= 0.3 \times 0.75(433 - 313) - 30 = -3\ kJ$$

Hence, heat 'rejected 'during the process

$Q = 3kJ$

Example 8: ***Air enters a compressor at $1\ Bar$ and having a specific volume of $1.8\ m^3/kg$ and is compressed to $5 \times 10^5\ pa$ isothermally. Determine :***
(i) Work done
(ii) Change in internal energy
(iii) Heat transfer



Solution :

Initial pressure of air , $p_1 = 10^5$ Pa

Initial temperature of air, $T_1 = 25 + 273 = 298\ K$

Final pressure of air, $p_2 = 5 \times 10^5\ Pa$

Final temperature of air, $T_2 = T_1 = 298\ K$ (Isothermal process)

Since, it is a closed steady state process, we can write down the first law of thermodynamics as,

$$Q = (u_2 - u_1) + W \quad \ldots\ldots per\ unit\ mass$$

(i) for isothermal process

$$W_{1-2} = \int_1^2 p.dv = p_1 v_1 ln\frac{p_1}{p_2}$$

$$p_1 v_1 = p_2 v_2 \quad for\ isothermal\ process$$

$$W_{1-2} = 10^5 \times 1.8\ ln\left(\frac{1 \times 10^5}{5 \times 10^5}\right)$$

$$= -2.897 \times 10^5 J/kg = -289.7\ kJ/kg$$

(- ve sign indicates that the work is supplied to the air (i.e system)

$$\therefore work\ done\ on\ the\ air = 289.7\ kJ/kg$$

(ii) since temperature is constant

$$u_2 - u_1 = 0$$

$$\therefore Change\ in\ internal\ energy = zero$$

(iii) $Again,\ Q_{1-2} = (u_2 - u_1) + W$

$= 0 + (-289.7) = -289.7$ KJ

(- ve sign indicates that heat is lost (rejects) from the system to the surroundings)

$$\therefore Heat\ transfeered = 289.7\ kJ/kg$$

Example 8: Air at 1.02 bar, 22^oC, initially occupying a cylinder volume of 0.015 m^3, is compressed reversibly and adiabatically by a piston to a pressure of 6.8 bar. Calculate the final temperature, the final volume, and the work done on the mass of air in the cylinder.

Given $\gamma = 1.4, R = 0.287 \frac{kJ}{kg.K}$

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$$

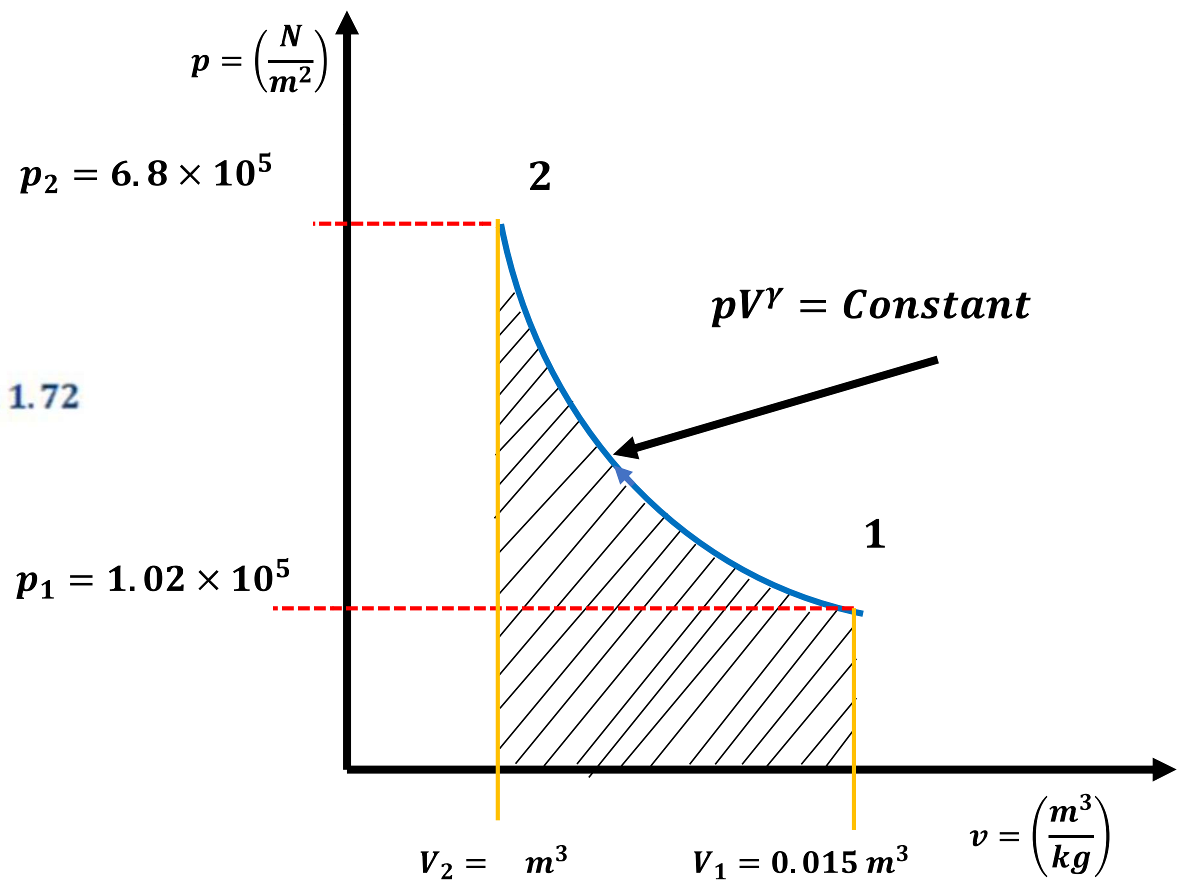
$$or, \quad T_2 = T_1 \times \left(\frac{p_2}{p_1}\right)^{\gamma-1/\gamma}$$

i.e.

$$T_2 = 295 \times \left(\frac{6.8}{1.02}\right)^{(1.4-1)/1.4} = 295 \times 6.67^{0.286} = 295 \times 1.72$$
$$= 507.5K$$

$$Where\ T_1 = 22 + 273 = 295\ K$$

i.e Final temperature, $T_2 = 507.5 - 273 = 234.5\ °C$



From Eqn. (2.25) :

$$\frac{p_2}{p_1} = \left(\frac{v_1}{v_2}\right)^{\gamma} \quad or \quad \frac{v_1}{v_2} = \left(\frac{p_2}{p_1}\right)^{1/\gamma}$$

$$\frac{0.015}{v_2} = \left(\frac{6.8}{1.02}\right)^{1/1.4} = 6.67^{0.714} = 3.87$$

$$\therefore v_2 = \frac{0.015}{3.87} = 0.00388\ m^3$$

$$i.e, \quad Final\ volume = 0.00388\ m^3$$

$$From\ Eqn. 2.21: \quad W = u_1 - u_2 = c_v(T_1 - T_2) = 0.718(295 - 507.5)$$
$$= -152.8\ kJ/kg$$

i.e, Work input per kg = 152.8 kJ/kg

The mass of air can be found using equation

$$PV = mRT$$

$$\therefore m = \frac{p_1 v_1}{RT} = \frac{1.02 \times 10^5 \times 0.015}{0.287 \times 10^3 \times 295} = 0.0181\ kg$$

i.e, Total work done = 0.0181× 152.8 =2.76 KJ