Chapter Two

The First Law of Thermodynamics

2.1 Introduction

When a system undergoes a thermodynamic cycle then the net heat supplied to the system from its surroundings is equal to the net work done by the system on its surroundings.

In symbols,

$$\oint dQ = \oint dW \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (2.1)$$

Where $\oint$ represent the integral for a complete cycle.

Q= heat energy transfer , W= work energy transfer

Or , can be written in this form , $\sum dQ = \sum dW$

2.2 sign convention of heat and work

For work:

1. if the work is done by the system on the surroundings, e.g., when a fluid expands pushing a piston outwards, the work is said to be positive.

i.e., Work output of the system +W

2. If the work is done on the system by the surroundings, e.g., when a force is applied to a rotating handle, or to a piston to compress a fluid, the work is said to be negative.

i.e., work input to system =-W

For Heat :

1. heat received by the system =+Q

2. Heat rejected or given up by the system = -Q

Comparison of work and heat

Similarities:

(i) Both are path functions and inexact differentials

(ii) Both are boundary phenomenon i.e., both are recognized at the boundaries of the system as they cross them

(iii) Both are associated with a process, not a state. Unlike properties, work and heat has no meaning at a state.

(iv) System take energy, but not work or heat.

Dissimilarities:

(i) in heat transfer temperature difference is required.

(ii) in a stable system there cannot be work transfer, however, there is no limitation for the transfer of heat.

Example:

In a certain steam plant the turbine develops 1000kW. The heat supplied to the steam in the boiler is 2800 KJ/kg , the heat rejected by the system to cooling water in the condenser is 2100 KJ/kg and the feed pump work required to pump the condensate back into the boiler is 5 KW. Calculate the steam flow round the cycle in kg/s.

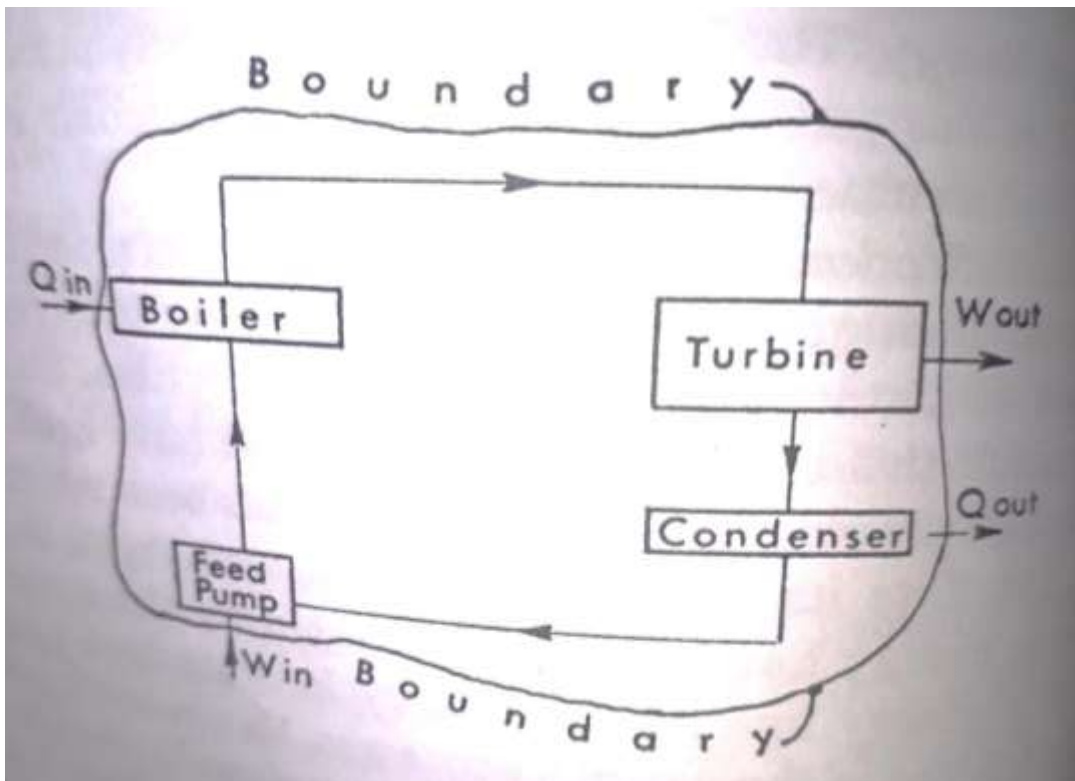**Solution : the cycle is shown diagrammatically in fig. 2.1.**

Fig. 2.1

Given :

Work output from the turbine (from the system)=W_T = 1000 KW

Heat supplied to the steam in the boiler (system) =Q$_B$ = 2800 KJ/kg

Heat rejected by the steam(system) from the condenser =Q$_C$= 2100 KJ/kg

Work input to the pump=W$_P$ = 5 KW

From the conservation of energy (first law of thermodynamics) equation:

$$\sum dQ = \sum dW \dots\dots\dots\dots\dots\dots (1)$$

$$\sum dQ = Q_B - Q_C = 2800 - 2100 = 700 \ KJ/kg$$

$$\sum dW = W_T - W_P = 1000 - 5 = 995 \ KW$$

Substitute in Eqn. (1) , we get

$$700 \times \overset{\circ}{m} = \dot{995}$$

$\overset{\circ}{m}$=1.421 kg/s

2.2 Corollaries of First Law of Thermodynamics

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The first law of thermodynamics, in the form of equation (2.1) , has a number of important consequences(or values) which are stated in the form of corollaries(or results):

2.2.1 Corollary 1

There exists a property of a closed system called energy such that a change in its value is equal to the difference between the heat supplied and the work done during any change of state. In other words, there exists a point function such that

$$\partial Q - \partial W = \Delta E = E_2 - E_1$$

Or, $Q - W = \Delta E = E_2 - E_1$ ....(2.2)

Energy E, is composed of internal energy,U , kinetic energy, K.E, and potential energy, P.E.

i.e.,   E = Internal energy + kinetic energy + Potential energy

or,   E = U + K.E + P.E

IN a closed system kinetic and potential energy effects generally play a negligible part, equation (2.2) may be simplified to :

$$Q = W + \Delta U \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots (2.3)$$

The property U is called the internal energy of the system . The property U is the outcome of the first law of thermodynamics. Physically, an increase in U is associated with a rise in temperature of the system.

2.2.2 Corollary 2, Conservation of energy

The internal energy of a closed system remains unchanged if the system is isolated from its surroundings.

Proof : By definition of an isolated system, Q=W=0, $\Delta U = U_2 - U_1 = 0$.

$$Q = 0, \quad W = 0 , \quad \Delta U = 0$$

2.2.3 Corollary 3, Perpetual motion machine of first kind

A perpetual motion machine of first kind ($pmmI$) is impossible.

A perpetual motion machine is a device which produces a continuous supply of work without requiring any interaction from surroundings or other systems. such a device would, in effect, create energy.

Equation (2.1) states, that, if a net amount of heat is not supplied during a cycle, no net amount of work can be obtained, that is

$$\oint \delta W = 0 \quad if \quad \delta Q = 0$$

The existence 0f a pmm would therefore violate the first law of thermodynamics.

Fig.2.2 : PMM1 kind
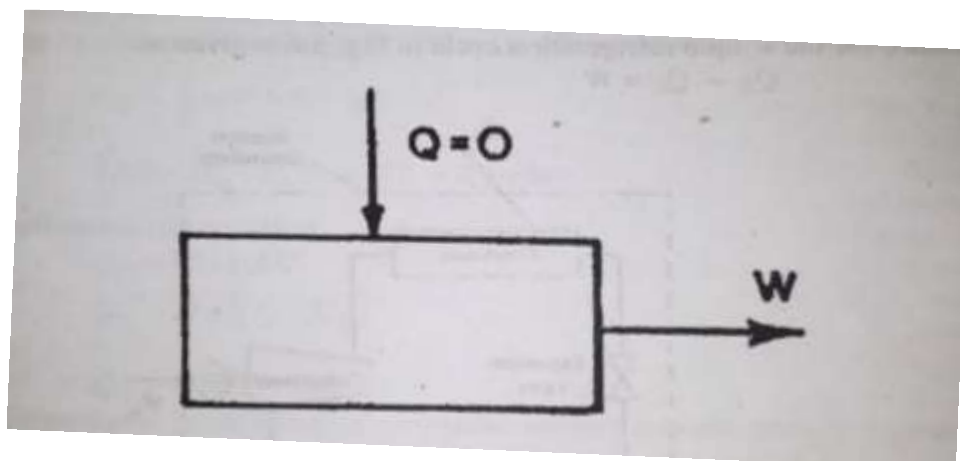
2.2.4 Energy balance

The basis of the first law of thermodynamics is the law of conservation of energy which, for a system, can be expressed as :

Net energy added to a system=Energy in – Energy out= increase in stored energy

But, for a system undergoing a cycle, there is no change in the state of the system so that the stored energy is zero. Hence :

Energy in = Energy out

2.3 The Perfect Gas

2.3.1 The Characteristic Equation of State

For ideal gas: $\dfrac{pv}{T} = CONSTANT = R$

The equation $\dfrac{pv}{T} = R$, is called the characteristic equation of a state of a perfect gas. The constant R is called the gas constant. Each perfect gas has a different gas constant.

Units of R are Nm/kg K or KJ/kg K.

Usually, the characteristic equation is written as

$pv = RT$ $\hspace{5cm}$ *(2.4)*

Or for m kg occupying V m^3

$PV = mRT$ $\hspace{5cm}$ *(2.5)*

- The characteristic equation in another form, can be derived by using kilogram-mole as a unit.

The kilogram-mole is defined as a quantity of a gas equivalent to M kg of gas, where M is the molecular weight of the gas (e.g., since the molecular weight of oxygen is 32, then 1 kg mole of oxygen is equivalent to 32 kg of oxygen).

$$m = nM \ldots\ldots\ldots\ldots\ldots\ldots. (2.6)$$

Where m=mass , n= number of mole , M= Molecular mass

Substituting for m from eqn.(2.6) in eqn.(2.5) gives

$$pV = nMRT$$

Or $\hspace{1.5cm} MR = \dfrac{pV}{nT}$

According to Avogadro's hypothesis the volume of 1 mole of any gas is the same as the volume of 1 mole of any other gas, when the gases are at the same temperature and pressure. Therefore $\frac{V}{n}$ is the same for all gases at the same value of p and T. That is the quantity $\frac{pV}{nT}$ is a constant for all gases. This constant is called universal gas constant, and is given the symbol, R_0.

i.e., $\quad\quad MR = R_0 = \frac{pV}{nT}$ $\quad\quad\quad\quad\quad\quad$ (2.7)

or, $\quad pV = nR_0T$

Since $MR = R_0$, then

$R = \frac{R_0}{M}$ $\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad$ (2.8)

It has been found experimentally that the volume of 1 mole of any perfect gas at 1 bar and $0°C$ is approximately 22.71 m³.

Therefore, from eqn.(2.7)

$R_0 = \frac{pV}{nT} = \frac{1×10^5×22.71}{1×(0°C+273)}$=8314.3 Nm/mole K

=8.3143 KJ/mole.K

Example : for oxygen which has a molecular weight of 32, the gas constant

$$R = \frac{R_0}{M} = \frac{8314}{32} = 259.8\, Nm/kg.K$$

2.3.2 Specific Heats

- the specific heat of a solid or liquid is usually defined as the heat required to raise unit mass through one degree temperature rise.

- for small quantities , we have

$dQ = mcdT$

Where m= mass , c=specific heat, and dT= temperature rise

Only two specific heats for gases are defined.

- *specific heat at constant volume, c_v*

-*specific heat at constant pressure, c_p*

And

We have $\quad dQ = mc_pdT$ for a reversible non-flow process at constant pressure……….(2.9)

And $dQ = mc_v dT$ for a reversible non-flow process at constant volume……..(2.10)

Integrating eqns. (2.9) and (2.10), we have

Flow of heat in a reversible constant pressure process

$$Q = mc_p(T_2 - T_1)………..(2.11)$$

Flow of heat in a reversible constant volume process

$$Q = mc_v(T_2 - T_1)………..(2.12)$$

In case of real gases, c_p and c_v vary with temperature. But s suitable average value may be used for most practical purpose.

2.3.3 Joule's Law

Joule's law states as follows:

" The internal energy of a perfect gas is a function of the absolute temperature only."

i.e., $u=f(T)$

To evaluate this function let 1 kg of a perfect gas be heated at constant volume

According to non-flow energy equation,

$$dQ = du + dW$$

$dW = 0$, since volume remains constant

$$\therefore Q = du$$

At constant volume for a perfect gas, from eqn.(2.12), for 1 kg

$$dQ = c_v dT$$

$$dQ = du = c_v dT$$

And integrating, $u = c_v T + K$, K being constant.

According to Joule's law :

u=0 for T=0 , hence constant K is zero.

i.e., internal energy, $u=c_v T$ for a perfect gas   (2.13)

or for mass m , of a perfect gas

internal energy, $U=m\ c_v T$…………………….(2.14)

Gain in internal energy,

$$U_2 - U_1 = mc_v(T_2 - T_1)\text{.....................................}(2.15)$$

Equation (2.15) gives the gains of internal energy for a perfect gas between two states for any process, reversible or irreversible.

2.3.4 Relationship Between two Specific Heats

Consider a perfect gas being heated at constant pressure from T_1 to T_2.

According to non-flow equation,

$$Q = (U_2 - U_1) + W$$

Also for a perfect gas,

$$U_2 - U_1 = mc_v(T_2 - T_1)$$

$$Q = mc_v(T_2 - T_1) + W$$

In a constant pressure process, the work done by the fluid,

$$W = P(V_2 - V_1)$$

$$= mR(T_2 - T_1)$$

$$\therefore P_1V_1 = mRT_1$$

$$P_2V_2 = mRT_2$$

$$P_1 = P_2 = P \text{ in this case}$$

On substituting

$$Q = mc_v(T_2 - T_1) + mR(T_2 - T_1)$$

But for constant pressure process,

$$Q = mc_p(T_2 - T_1)$$

By equating the two expression, we have

$$m(c_v + R)(T_2 - T_1) = mc_p(T_2 - T_1)$$

$$\therefore (c_v + R) = c_p$$

Or $\quad c_p - c_v = R$ ....(2.16)

Dividing both side by c_v , we get

$$\frac{c_p}{c_v} - 1 = \frac{R}{c_v}$$

$$\therefore \quad \gamma - 1 = \frac{R}{c_v}$$

Or , $c_v = \frac{R}{\gamma-1}$ (2.16a)

Where,

$$\gamma = \frac{c_p}{c_v}$$

Similarly, dividing both sides by $c_p, we\ get$

$$c_p = \frac{\gamma R}{\gamma - 1} \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (2.16b)$$

2.3.5 Enthalpy

In thermodynamics is the sum of internal energy(u) and pressure volume product(pv). This sum is called Enthalpy (h)

i.e.,   h=u + pv   ……………………….(2.17)

the total enthalpy of mass, m , of a fluid can be

H=U + PV   where H=m.h

For a perfect gas,

$$h = u + pv$$

$$h = c_v T + RT = (c_v + R)T = c_p T$$

$$i.e., \quad h = c_p T$$

$$H = mc_p T$$

2.3.6 Ratio of Specific Heats

The ratio of specific heat at constant pressure to the specific heat at constant volume is given by the symbol $\gamma(gamma)$.

$$\gamma = \frac{c_p}{c_v}$$

Since $c_p = c_v + R$, it is clear that c_p must be greater than c_v for any perfect gas. It follows , therefore, that the ratio,

$$\frac{c_p}{c_v} = \gamma \ \textit{is always greater than unity}.$$

2.4 Application of first law of thermodynamics to non-flow or closed system

1. Reversible constant volume process (v=constant)

In a constant volume process:

(i) the working substance is contained in a rigid vessel
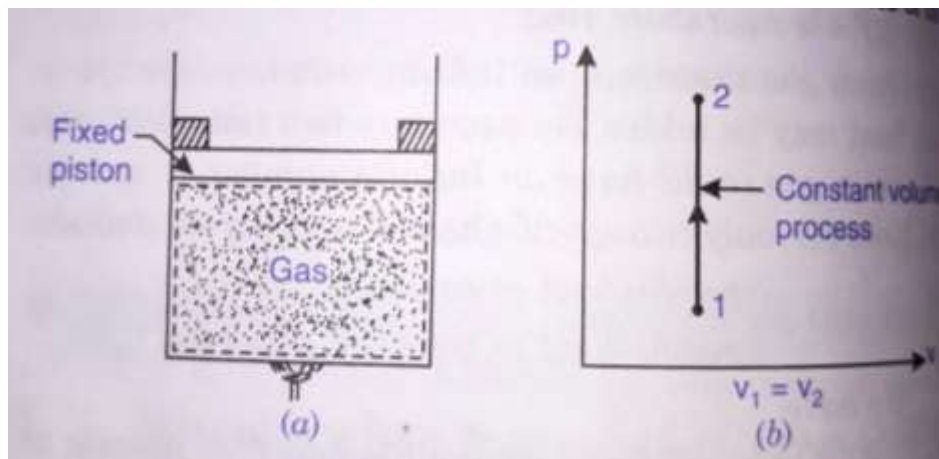
(ii) work=0 , since v=constant

Fig.2.3 : Reversible constant volume process

Considering mass of the working substance unity and applying first law of thermodynamics to the process

$$Q = (u_2 - u_1) + W \ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (2.18)$$

The work done $W = \int_1^2 pdv = 0 \ as \ dv = 0$

$$\therefore Q = (u_2 - u_1) = c_v(T_2 - T_1) \ \dots\dots\dots\dots\dots\dots\dots\dots\dots (2.18a) \quad (\because U = mu)$$

Where $c_v=$ **specific heat at constant volume.**

For mass, m , of working substance

$$Q = U_2 - U_1 = mc_v(u_2 - u_1)\dots\dots\dots\dots\dots\dots\dots\dots(2.18b)$$

2. Reversible constant pressure process (p=constant)

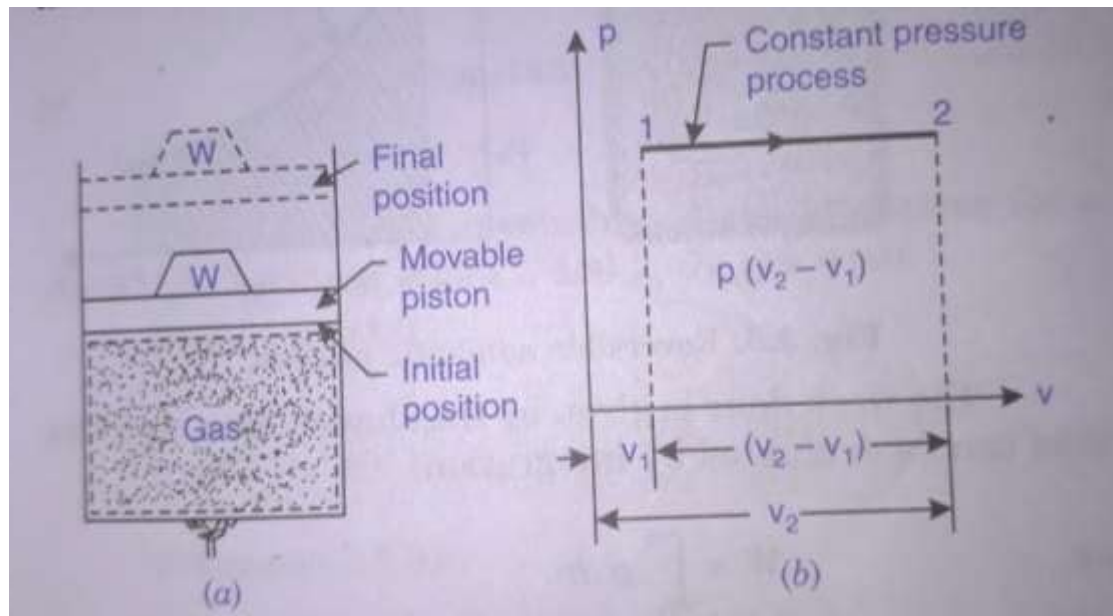Fig.2.4 shows the system and states before and after the heat addition at constant pressure.

Fig.2.4 Reversible constant pressure process

Considering unit mass of working substance and **Appling first law of thermodynamics to the process, i.e., Non-Flow Energy Equation(NFEE):**

$$Q = (u_2 - u_1) + W$$

The work done, $W = \int_1^2 p\, dv = p(v_2 - v_1) = pv_2 - pv_1$

$$\therefore Q = (u_2 + p_2 v_2) - (u_1 + p_1 v_1) = h_2 - h_1$$

$[\because p_1 = p_2 = p \quad , \quad h = u + pv \;]$

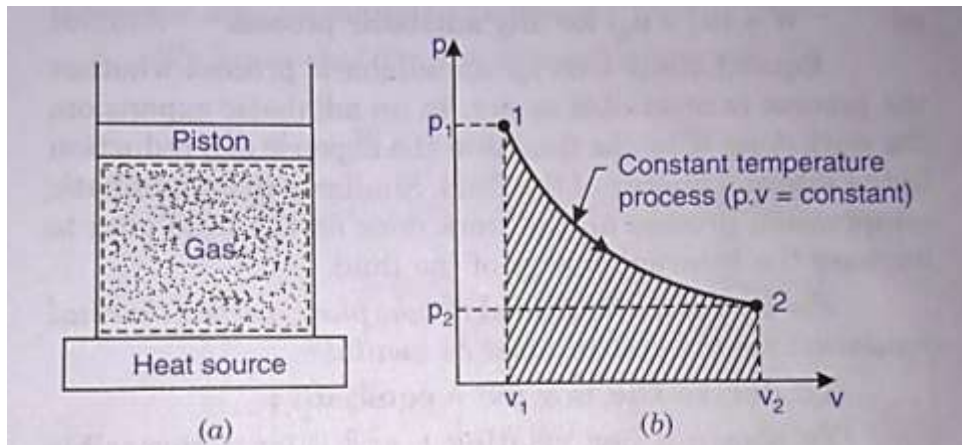$$Q = h_2 - h_1 = c_p(T_2 - T_1)$$

Where h= Enthalpy (specific), and

$$c_p = specific\ heat\ at\ constant\ pressure$$

For mass ,m, of working substance

$$Q = H_2 - H_1 = mc_p(T_2 - T_1) \dots \dots \dots \dots \dots \dots \dots \dots (2.19)$$

3. Reversible Temperature or Isothermal Process (pv=constant, T=constant)

A process at a constant temperature is called an isothermal process

Fig.2.6. Reversible isothermal process

Considering unit mass of working substance and applying first law to the process

$$Q = (u_2 - u_1) + W$$

$$= c_v(T_2 - T_1) + W$$

$$= 0 + W \qquad [\because T_1 = T_2]$$

The work done, $W = \int_1^2 pdv$

In this case $pv = c \quad or \quad p = c/v$

$$\therefore W = \int_{v_1}^{v_2} C\frac{dv}{v} = C[lnv]_{v_1}^{v_2} = Cln\frac{v_2}{v_1}$$

The constant C can either be written as p_1v_1 **or as** p_2v_2 **. since**

$$p_1v_1 = p_2v_2 = constanr, C$$

i. e, $\quad W = p_1v_1ln\frac{v_2}{v_1}$ **per unit mass of working substance**

$$\therefore \quad Q = W = p_1v_1ln\frac{v_2}{v_1} \qquad \dots\dots\dots\dots.(2-20)$$

For mass ,m, of working substance

$$\therefore \quad Q = W = p_1 V_1 ln \frac{V_2}{V_1} \qquad (2-20a)$$

For isothermal process : $p_1 V_1 = p_2 V_2$

$$\therefore \quad \frac{V_2}{V_1} = \frac{P_1}{P_2}$$

Or, $\quad Q = W = p_1 V_1 ln \frac{p_1}{p_2} \qquad \ldots \ldots \ldots (2-20b)$

4. Reversible Adiabatic Process $(pv^\gamma = constant)$

An adiabatic process is one in which no heat is transferred to or from the fluid(system) during the process (i.e Q=0).

Such a process can be reversible or irreversible. The reversible adiabatic non-flow process will be considered in this section.

Considering unit mass of working substance and applying first law to the process, or

Non-Flow Energy Equation (NFEE):

$$Q = (u_2 - u_1) + W$$

$$0 = (u_2 - u_1) + W$$

Or $\quad W = (u_1 - u_2) \quad for\ any\ adiabatic\ process \quad \ldots \ldots \ldots (2-21)$

For adiabatic expansion : +W

For adiabatic compression : -W

For an adiabatic process to take place, perfect thermal insulation for the system must be available.

To drive the general law for an adiabatic process : $pv^\gamma = constant$

To obtain a law relating p and v for a reversible adiabatic process let us consider the non-flow energy equation in differential form,

$$dQ = du + dW$$

For a reversible process

$$dW = pdv$$

$$\therefore dQ = du + pdv = 0$$

(since for an adiabatic process Q=0)

Also for a perfect gas

$$pv = RT \quad or \quad p = \frac{RT}{v}$$

Hence substituting,

$$du + \frac{RTdv}{v} = 0$$

Also $\quad u = c_v T \quad or \quad du = c_v dT$

$$\therefore \quad c_v dT + \frac{RTdv}{v} = 0$$

Dividing both sides by T, we get

$$c_v \frac{dT}{T} + \frac{Rdv}{v} = 0$$

Integrating

$$c_v lnT + Rln\, v = constant$$

Substituting $\quad T = \frac{pv}{R}$

$$c_v ln\frac{pv}{R} + Rln\, v = constant$$

Dividing throughout both sides by c_v

$$ln\frac{pv}{R} + \frac{R}{c_v}.ln\, v = constant$$

Again : $c_v = \frac{R}{(\gamma - 1)} \quad or \quad \frac{R}{c_v} = \gamma - 1$

Hence substituting

$$ln\frac{pv}{R} + (\gamma - 1)ln\, v = constant$$

$$ln\frac{pv}{R} + ln\, v^{\gamma-1} = constant$$

$$ln\frac{pv \times v^{\gamma-1}}{R} = constant$$

$$ln\frac{pv \times v^{\gamma} \times v^{-1}}{R} = constant$$

$$ln\frac{p.v^{\gamma}}{R} = constant$$
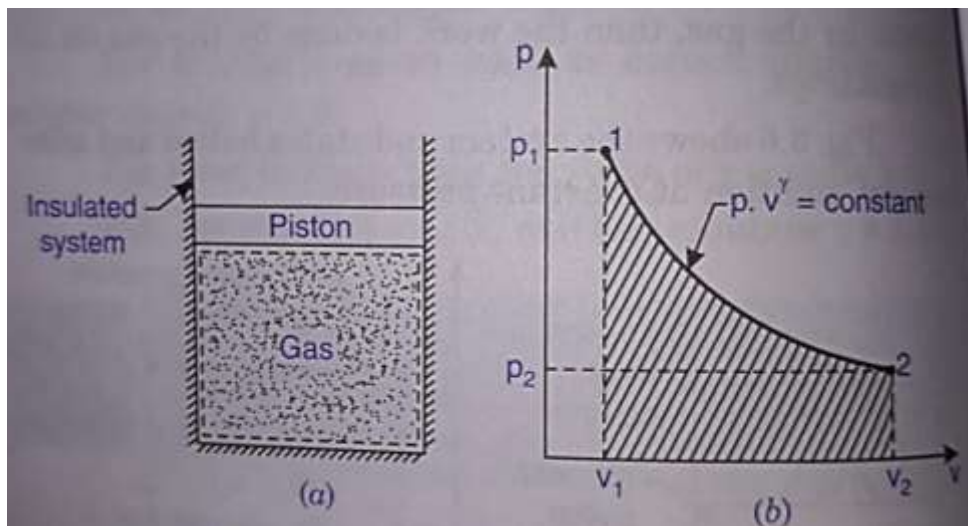
$$\frac{pv^\gamma}{R} = e^{constant} = constant$$

Since R = gas constant=constant

$$pv^\gamma = R.\,constant = constant$$

Or $pv^\gamma = constant$

Expression for Work W :

A reversible adiabatic process for a perfect gas is shown on a $p - v\ diagram$ in Fig. 2-7 :

(a)   (b)

Fig. 2.7 : Reversible adiabatic process

The work done is given by the shaded area, and this area can be evaluated by integration.

i.e, $W = \int_{v_1}^{v_2} p\,dv$

Therefore, since $pv^\gamma = constant$, C , then $[\because p = \frac{C}{v^\gamma}]$

$$W = \int_{v_1}^{v_2} C\,\frac{dv}{v^\gamma}$$

i.e , $W = C \int_{v_1}^{v_2} \frac{dv}{v^\gamma} = C \left|\frac{v^{-\gamma+1}}{-\gamma+1}\right|_{v_1}^{v_2}$

$$= C \left(\frac{v_2^{-\gamma+1} - v_1^{-\gamma+1}}{-\gamma + 1} \right) = C \left(\frac{v_1^{-\gamma+1} - v_2^{-\gamma+1}}{\gamma - 1} \right)$$

The constant in this equation can be written as $p_1 v_1^{\gamma}$ or as $p_2 v_2^{\gamma}$. Hence

$$W = \frac{p_1 v_1^{\gamma} v_1^{-\gamma+1} - p_2 v_2^{\gamma} v_2^{-\gamma+1}}{\gamma - 1} = \frac{p_1 v_1 - p_2 v_2}{\gamma - 1}$$

i.e , $\quad W = \frac{p_1 v_1 - p_2 v_2}{\gamma - 1} \quad \dots \dots \dots \dots \dots (2-22)$

Relationship between T and V and T and p :

By using equation $pv = RT$, the relationship between T and v , and T and p, may by derived as follows :

i.e , $\quad pv = RT$

$$\therefore p = \frac{RT}{v}$$

Putting this value in the equation $pv^{\gamma} = constant, C$

$$\frac{RT}{v} . v^{\gamma} = constant, C$$

$$\frac{T}{v} . v^{\gamma} = \frac{C}{R} = constant$$

$$Tv^{\gamma-1} = constant \dots \dots \dots \dots (2-23)$$

From eqn. (2.23) :

$$T_1 v_1^{\gamma-1} = T_2 v_2^{\gamma-1}$$

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2} \right)^{\gamma-1} \dots \dots \dots (2.24)$$

For an adiabatic process :

$$p_1 v_1^{\gamma} = p_2 v_2^{\gamma}$$

$$i.e , \quad \frac{p_2}{p_1} = \left(\frac{v_1}{v_2} \right)^{\gamma} \dots \dots \dots (2.25)$$

Also from Eqn.(2.25) :

$$\frac{v_1}{v_2} = \left(\frac{p_2}{p_1}\right)^{\frac{1}{\gamma}} \quad \dots \dots (2.26)$$

Substituting Eqn.(2.26) in Eqn. (2.24):

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} \dots \dots \dots (2.27)$$

Hence for an adiabatic process :

$$(\textbf{Temperature Ratio}) = (\textbf{Inverse Volume Ratio})^{\gamma-1}$$
$$= (\textbf{Pressure Ratio})^{\frac{\gamma-1}{\gamma}}$$

Or ,

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{\gamma-1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} \dots \dots \dots (2.28)$$

From eqn. (2.21) , the work done in an adiabatic process per kg of gas is given by

$$W = (u_1 - u_2)$$

$$W = c_v(T_1 - T_2)$$

Also , we know that

$$c_v = \frac{R}{\gamma - 1}$$

Hence, substituting, we get

$$W = \frac{R(T_1 - T_2)}{\gamma - 1}$$

Using equation $\quad pv = RT$

$$W = \frac{(p_1 v_1 - p_2 v_2)}{\gamma - 1}$$

This is the same expression obtained before as eqn. (2.22)

5. Polytropic Reversible Process (pv^n=constant)

The general law of polytropic (general) process is
$pv^n = constant, \ where , n \ is \ a \ polytropic \ index , n \ is \ constant$

We know that for any reversible process

$$W = \int p\, dv$$

For a process in $pv^n = constant, we\ have$

$$p = \frac{C}{v^n}, where\ C\ is\ a\ constant$$

$$\therefore W = C \int_{v_1}^{v_2} \frac{dv}{v^n} = C\left.\frac{v^{-n+1}}{-n+1}\right|_{v_1}^{v_2} = C\left(\frac{v_2^{-n+1} - v_1^{-n+1}}{-n+1}\right)$$

$$W = C\left(\frac{v_1^{-n+1} - v_2^{-n+1}}{n-1}\right)$$

Since the constant C can be written as $p_1 v_1^n$ *or as* $p_2 v_2^n$

Substituting for C

$$W = \frac{p_1 v_1^n v_1^{-n+1} - p_2 v_2^n v_2^{-n+1}}{n-1}$$

i.e, the work done, $\quad W = \dfrac{p_1 v_1 - p_2 v_2}{n-1}$ ..... (2.29)

$or,\quad W = \dfrac{R(T_1 - T_2)}{n-1}$ ...... (2.30)

It follows also that for any polytropic process , we can write

$$\frac{p_2}{p_1} = \left(\frac{v_1}{v_2}\right)^n \dots\dots\dots (2.31)$$

The following relations can be derived (following the same procedure as was done under reversible adiabatic process)

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{n-1} \dots\dots\dots (2.32)$$

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} \dots\dots\dots (2.33)$$

Heat transfer during polytropic process (for perfect gas pv=RT) :

Using non-flow energy equation, the heat flow/transfer during the process can be found,

$$Q = (u_2 - u_1) + W$$

$$= c_v(T_2 - T_1) + \frac{R(T_1 - T_2)}{n - 1}$$

$$Q = \frac{R(T_1 - T_2)}{n - 1} - c_v(T_1 - T_2)$$

Also

$$c_v = \frac{R}{(\gamma - 1)}$$

On substituting

$$Q = \frac{R}{n - 1}(T_1 - T_2) - \frac{R}{(\gamma - 1)}(T_1 - T_2)$$

$$Q = R(T_1 - T_2)\left(\frac{1}{n - 1} - \frac{1}{\gamma - 1}\right)$$

$$= \frac{R(T_1 - T_2)(\gamma - 1 - n + 1)}{(\gamma - 1)(n - 1)} = \frac{R(T_1 - T_2)(\gamma - n)}{(\gamma - 1)(n - 1)}$$

$$\therefore Q = \frac{(\gamma - n)R(T_1 - T_2)}{(\gamma - 1)(n - 1)}$$

Or

$$Q = \left(\frac{\gamma - n}{\gamma - 1}\right).W \qquad \left[\because W = \frac{R(T_1 - T_2)}{(n - 1)}\right] \dots\dots\dots (2.34)$$

In a polytropic process, the **index n** depends only on the **heat and work quantities** during the process.

....

The various processes considered earlier are special cases of polytropic process for a perfect gas. For example

(i) when n=0 $\quad pv^0 = constant$ *i.e, p = constant : reversible constant pressure process*

(ii) when n= ∞ $pv^∞ = constant$

$\sqrt[∞]{pv^∞} = constant$

$p^{\frac{1}{∞}}.v^{\frac{∞}{∞}} = constant$

$p^0.v = constant$

i.e,   v=constant : reversible constant volume process

(iii) when n=1   pv=constant i.e   T=constant , since
$pv/T = constant\ for\ gas : reversible\ isothermal\ process$

(iv) when n = γ,
$pv^γ = constant, i, e, reversible\ adiabatic\ process$

(v) when n=n , $pv^n = constant$, reversible polytropic process

This is illustrated on a p-v diagram in Fig. 2.8.

(i) state 1 to state A is *constant pressure* cooling (n=0)

(ii) state 1 to state B is *isothermal compression* (n=1)

(iii) state 1 to state C is *reversible adiabatic compression* $(n = γ)$

(iv) state 1 to state D is *constant volume heating* $(n = ∞)$

Similarly,
(i) state 1 to state Á is *constant pressure heating* (n=0)

(ii) state 1 to state B́ is *isothermal expansion* (n=1).

(iii) state 1 to state Ć is *reversible adiabatic expansion* $(n = γ)$.

(iv) state 1 to D́ is *constant volume cooling* $(n = ∞)$
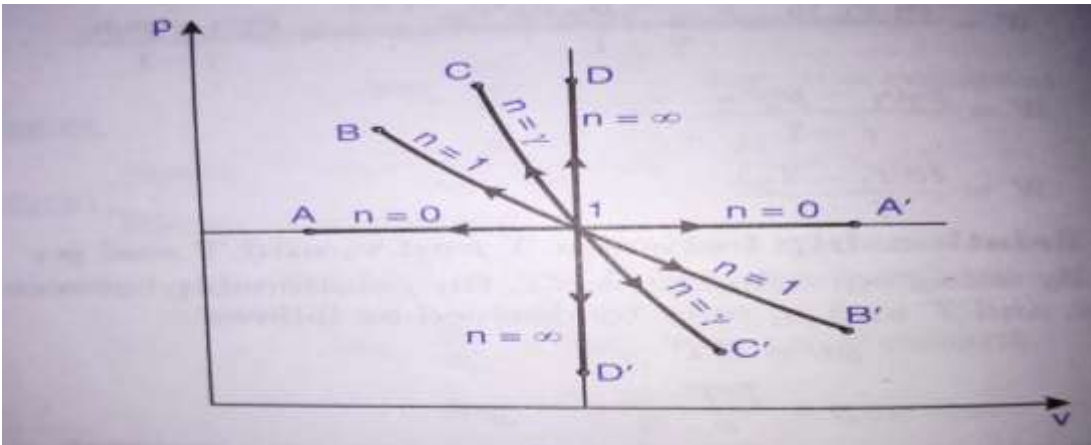
Fig. 2.8.

2-3 Free Expansion

　　Consider two vessels 1 and 2 interconnected by a short pipe with a valve A, and perfectly thermally insulated (Fig.2.9). initially let the vessel 1 be filled with a fluid at a certain pressure, and let 2 be completely evacuated. When the valve A is opened the fluid in 1 will expaned rapidly to fill both vessels 1 and 2 . the pressure finally will be lower than the initial pressure in vessel 1 .this is known as *free or unresisted expansion..* The process is highly irreversible.

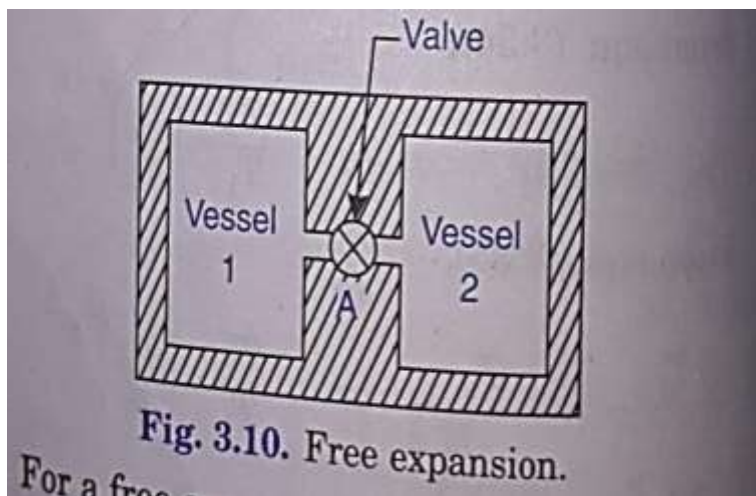Fig. 3.10. Free expansion.

For a free

Fig. 2.9 : Free Expansion

Now applying first law of thermodynamics (or Non-Flow Energy Equation, NFEE) between the initial and final states,
$$Q = (u_2 - u_1) + W$$
In this process, no work is done on or by the fluid, since the boundary of the system does not move. No heat flows to or from the fluid since the system is well insulated. The process is therefore, *adiabatic* but *irreversible.*
$$i.e, \quad u_2 - u_1 = 0 \quad or \quad u_2 = u_1$$

In a free expansion, therefore,
The internal energy initially= the initial energy finally
For perfect gas

$$u = c_v T$$
$$\therefore \textit{ For free expansion of a perfect gas,}$$
$$c_v T_1 = c_v T_2$$
$$i.e \quad T_1 = T_2$$

Table 2.1. Summary of Processes for Perfect Gas (Unit mass)

Process	Index n	Heat added	$\int_1^2 pdv$	p, v, T relation	Specific heat, c
Constant pressure	n=0	$c_p(T_2 - T_1)$	$p(v_2 - v_1)$	$\dfrac{T_2}{T_1} = \dfrac{v_2}{v_1}$	c_p
Constant volume	$n = \infty$	$c_v(T_2 - T_1)$	0	$\dfrac{T_1}{T_2} = \dfrac{p_1}{p_2}$	c_v
Constant temperature	$n = 1$	$p_1 v_1 ln\dfrac{v_2}{v_1}$	$p_1 v_1 ln\dfrac{v_2}{v_1}$	$p_1 v_1 = p_2 v_2$	∞
Reversible adiabatic	$n = \gamma$	0	$\dfrac{p_1 v_1 - p_2 v_2}{\gamma - 1}$	$p_1 v_1{}^\gamma = p_2 v_2{}^\gamma$ $\dfrac{T_2}{T_1} = \left(\dfrac{v_1}{v_2}\right)^{\gamma - 1}$ $= \left(\dfrac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$	0
Polytropic process	$n = n$	$= \dfrac{\gamma - n}{\gamma - 1}$ $\times W_{polytropic}$	$\dfrac{p_1 v_1 - p_2 v_2}{n - 1}$	$p_1 v_1{}^n = p_2 v_2{}^n$ $\dfrac{T_2}{T_1} = \left(\dfrac{v_1}{v_2}\right)^{n - 1}$ $= \left(\dfrac{p_2}{p_1}\right)^{\frac{n-1}{n}}$	c_n $= c_v \left(\dfrac{\gamma - n}{1 - n}\right)$

Example 2.1

2 kg of an ideal gas with constant specific heats begins a process at 200 kPa and 60℃. The gas is first expanded at constant pressure until its volume doubles. Then it is heated at constant volume until its pressure doubles. Calculate the work done by the gas during the entire process and the final temperature. Take, R= 0.6 kJ/kg.K and $c_p = 2$ kJ/kg.K

Solution :
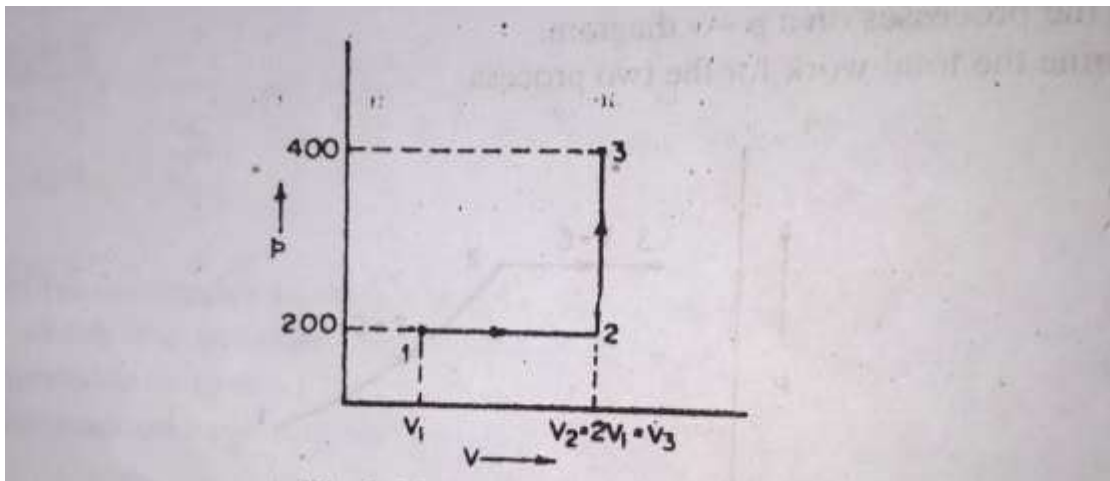
Fig.2.5 p-v diagram

Using the gas equation

$$p_1V_1 = mRT_1 \qquad\qquad T_1 = 60 + 273 = 333 \ K$$

$$200 \times V_1 = 2 \times 0.6 \times (333)$$

$$V_1 = 1.998 \ m^3$$

$$V_2 = 2V_1 = 2 \times 1.998 = 3.996 m^3$$

$$w_{12} = p_1(V_2 - V_1) = 200(3.996 - 1.998 = 399.6 \ kJ$$

$$w_{23} = 0 \ since \ for \ constant \ volume \ process \ dv = 0$$

$$w_{net} = w_{12} + w_{23} = 399.6 + 0 = 399.6 \ kJ$$

$$p_3V_3 = mRT_3$$

$$400 \times 3.996 = 2 \times 0.6T_3$$

$$T_3 = 1332K$$

H.W:

1) Calculate Q$_{12}$

2) Calculate Q$_{23}$

3) Calculate Q$_{net}$

الحل للواجب البيتي **H.W:**

Solution of H.W :

For gas : $p_2V_2 = mRT_2$

$$200 \times 3.996 = 2 \times 0.6 \times T_2$$

$$T_2 = 666 \ K$$

For constant pressure process 12, heat transfer during this process is :

$$Q_{12} = mc_p(T_2 - T_1) = 2 \times 2(666 - 333) = 1332KJ$$

$$R = c_p - c_v$$

$$c_v = c_p - R = 2 - 0.6 = 1.4 \ KJ/kg.K$$

For constant volume process 23, heat transfer during this process is :

Since W=0 , for constant volume process, hence

$$Q_{23} = \Delta U = U_3 - U_2 = mc_v(T_3 - T_2) = 2 \times 1.4(1332 - 666) = 1864.8 \ KJ$$

$$Q_{net} = Q_{12} + Q_{23} = 1332 + 1864.8 = 3196.8KJ$$

Example 2.2 .

 In an internal combustion engine, during the compression stroke the heat rejected to the cooling water is $50 \ kJ/kg$ *and the work input is* $100 \ kJ/kg$ *. Calculate the change in internal energy of the working fluid stating whether it is a gain or loss*

Solution:

Heat rejected to the cooling water,

$$Q = -50 \ kJ/kg$$

(-ve sign since heat is rejected)

Work input,   W= -100 kJ/kg

(-ve sign since work is supplied to the system)

Using the relation,

$$Q = (u_2 - u_1) + W$$
$$-50 = (u_2 - u_1) - 100$$

Or

$$u_2 - u_1 = -50 + 100 = 50 \ kJ/kg$$

Hence , gain in internal energy = $50 \ kJ/kg$

Example 2.3.

 0.3 kg of nitrogen gas at 100 kPa and $40 \ ℃$ *is contained in a cylinder. The piston is moved compressing nitrogen until the pressure becomes 1 MPa and temperature becomes* $160 \ ℃$ *. The work done during the process is 30 kJ. Calculate the heat transferred from the nitrogen (i.e system) to the surrounding. Given* c_v *for nitrogen = 0.75 kJ/kg.K*
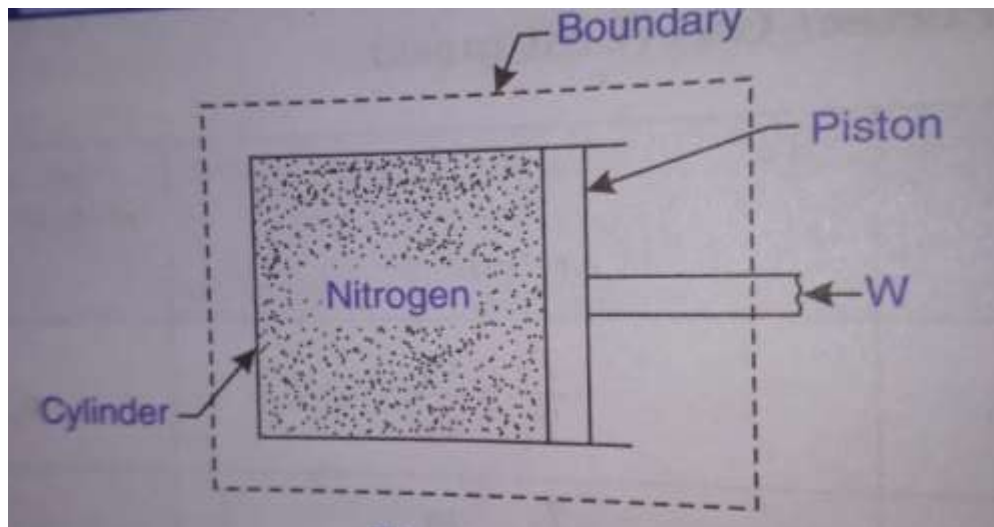
Solution :

Mass of nitrogen, m=0.3 kg

Temperature before compression = $40 \ ℃ = (40 + 273 = 313 \ K)$

Temperature after compression = $160 \ ℃ = (160 + 273 = 433 \ K)$

The work done during the compression process,

W= -30 KJ

Fig.2.10

According to first law of thermodynamics.

$$Q = \Delta U + W = (U_2 - U_1) + W$$
$$= mc_v(T_2 - T_1) + W$$

$$= 0.3 \times 0.75(433 - 313) - 30 = -3 \, kJ$$

Hence, heat 'rejected' during the process

$$Q = 3kJ$$

Example 2.4.

Air enters a compressor at 10^5 Pa and $25 \, °C$ having a volume of $1.8 \, m^3/kg$ and is compressed to $5 \times 10^5 \, Pa$ isothermally . Determine :

(i) Work done
(ii) Change in internal energy
(iii) Heat transfer

Solution :

Initial pressure of air , $p_1 = 10^5$ Pa

Initial temperature of air, $T_1 = 25 + 273 = 298 \, K$

Final pressure of air, $p_2 = 5 \times 10^5 \, Pa$

Final temperature of air, $T_2 = T_1 = 298 \, K$ (Isothermal process)

Since, it is a closed steady state process, we can write down the first law of thermodynamics as,

$$Q = (u_2 - u_1) + W \quad \text{.....} per \, unit \, mass$$

(i) for isothermal process

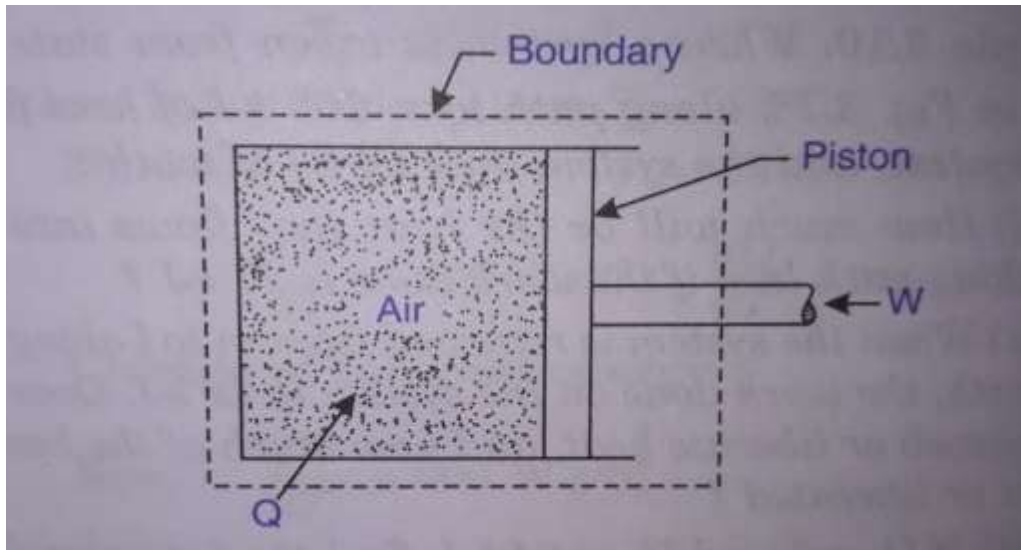$$W_{1-2} = \int_1^2 p.\, dv = p_1 v_1 \ln\frac{p_1}{p_2}$$

Fig.2.11

As

$$p_1 v_1 = p_2 v_2 \quad for\ isothermal\ process$$

$$W_{1-2} = 10^5 \times 1.8\ ln\left(\frac{1 \times 10^5}{5 \times 10^5}\right)$$

$$= -2.897 \times 10^5\ J/kg = -289.7\ kJ/kg$$

(- ve sign indicates that the work is supplied to the air (i.e system)

$$\therefore work\ done\ on\ the\ air = 289.7\ kJ/kg$$

(ii) since temperature is constant

$$u_2 - u_1 = 0$$

$$\therefore Change\ in\ internal\ energy = zero$$

(iii) *Again,* $Q_{1-2} = (u_2 - u_1) + W$

$= 0 + (- 289.7) = -289.7$ KJ

(- ve sign indicates that heat is lost (rejects) from the system to the surroundings)

$$\therefore Heat\ transfeered = 289.7\ kJ/kg$$

Example 2.5
Air at 1.02 bar, 22°C, initially occupying a cylinder volume of 0.015 m³, is compressed reversibly and adiabatically by a piston to a pressure of 6.8 bar. Calculate the final temperature, the final volume, and the work done on the mass of air in the cylinder. Given γ
$for\ air = 1.4\ , R = 0.287\ kJ/kg.K$
Solution :
From Eqn. (2.27):

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$or, \quad T_2 = T_1 \times \left(\frac{p_2}{p_1}\right)^{\gamma-1/\gamma}$$

i.e.

$$T_2 = 295 \times \left(\frac{6.8}{1.02}\right)^{(1.4-1)/1.4} = 295 \times 6.67^{0.286} = 295 \times 1.72$$

$$= 507.5K$$

$$Where\ T_1 = 22 + 273 = 295\ K$$

i.e Final temperature, $T_2 = 507.5 - 273 = 234.5\ °C$

From Eqn. (2.25) :

$$\frac{p_2}{p_1} = \left(\frac{v_1}{v_2}\right)^{\gamma} \quad or \quad \frac{v_1}{v_2} = \left(\frac{p_2}{p_1}\right)^{1/\gamma}$$

$$\frac{0.015}{v_2} = \left(\frac{6.8}{1.02}\right)^{1/1.4} = 6.67^{0.714} = 3.87$$

$$\therefore v_2 = \frac{0.015}{3.87} = 0.00388\ m^3$$

$$i.e, \quad Final\ volume = 0.00388\ m^3$$

$From\ Eqn.\ 2.21: \quad W = u_1 - u_2 = c_v(T_1 - T_2) = 0.718(295 - 507.5)$
$$= -152.8\ kJ/kg$$

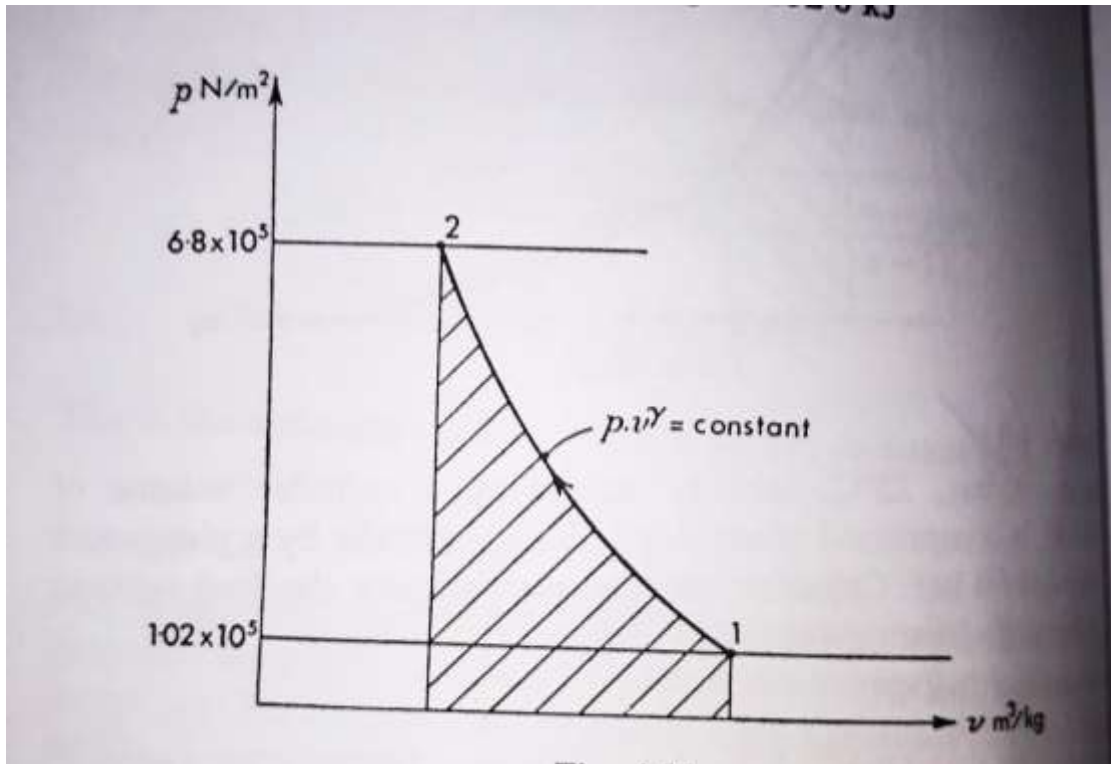i.e, **Work input per kg = 152.8** kJ/kg

Fig.2.12

The mass of air can be found using equation

$$PV = mRT$$

$$\therefore m = \frac{p_1 v_1}{RT} = \frac{1.02 \times 10^5 \times 0.015}{0.287 \times 10^3 \times 295} = 0.0181 \; kg$$

i.e, Total work done = 0.0181× 152.8 =2.76 KJ

The process is shown on a p-v diagram in fig. 2.12, the shaded area representing the work done per kg of air.