4. Balancing of Rotating Masses

4.1 Introduction

The high speed of engines and other machines is a common phenomenon now-a-days. It is, therefore, very essential that all the rotating and reciprocating parts should be completely balanced as far as possible. If these parts are not properly balanced, the dynamic forces are set up. These forces not only increase the loads on bearings and stresses in the various members, but also produce unpleasant and even dangerous vibrations. In this chapter we shall discuss the balancing of unbalanced forces caused by rotating masses, in order to minimize pressure on the main bearings when an engine is running.

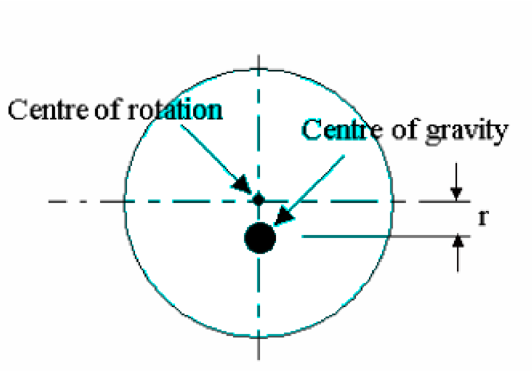
4.2 Balancing of Rotating Masses

The balancing of rotating bodies is important to avoid vibrations. In heavy industrial machines such as steam turbines and electric generators, vibration could cause catastrophic failure. Vibrations are noisy and uncomfortable and when a car wheel is out of balance, the ride is quite unpleasant. In the case of a simple wheel, balancing simply involves moving the center of gravity to the center of rotation but as we shall see. for longer and more complex bodies, there is more to it. For a body to be completely balanced it must have two things.

1. ***Static Balance***. This occurs when there is no resultant centrifugal force and the center of gravity is on the axis of rotation.
2. ***Dynamic Balance***. This occurs when there is no resulting turning moment alone the axis.

4.3 Balancing in one plane

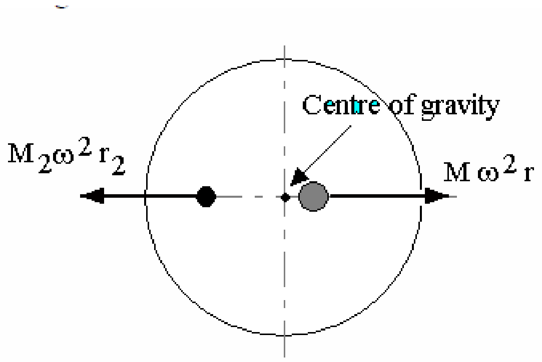
If the system is a simple disc then static balance is all that is needed. Consider a thin disc or wheel on which the center of gravity is not the same as the center of rotation. A simple test for static balance is to place the wheel in frictionless bearings. The center of gravity will always come to rest below the center of rotation (like a pendulum). If it is balanced it will remain stationary no matter which position it is turned to.



If the center of gravity is distance ***r*** from the center of rotation then when it spins at ***ω*** rad/s. centrifugal force is produced. This has a formula

$$F_C = m \cdot \omega^2 \cdot r$$

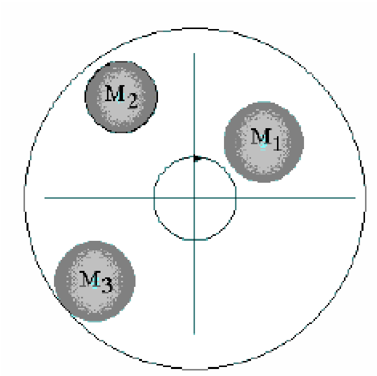
where ***m*** is the mass of the disc. This is the out of balance force. In order to cancel it out an equal and opposite force is needed. This is simply done by adding a mass m_2 at a radius r_2 as shown. The two forces must have the same magnitudes.



$$m \cdot \omega^2 \cdot r = m_2.\omega^2 \cdot r_2 \; or \; m \cdot r = m_2. \cdot r_2$$

Placing a suitable mass at a suitable radius moves the **center** of gravity to the center of rotation. This balance holds true at all speeds down to zero hence it is balanced so long as the products of ***m*** and ***r*** are equal and opposite.

Now consider that our disc is out of balance because there are three masses attached to it as shown. The 3 masses are said to be coplanar and they rotate about a common centre.



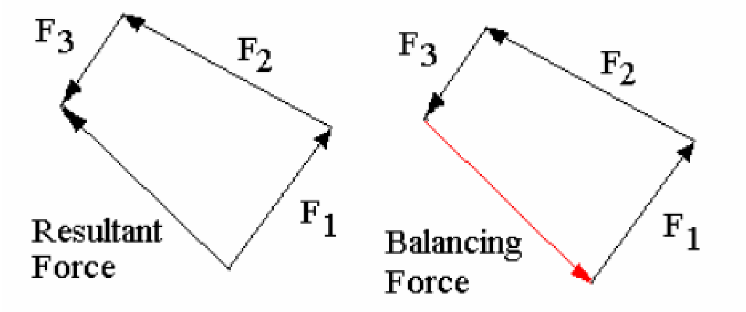
The centrifugal force acting on each mass is

$$F_C = m \cdot \omega^2 \cdot r$$

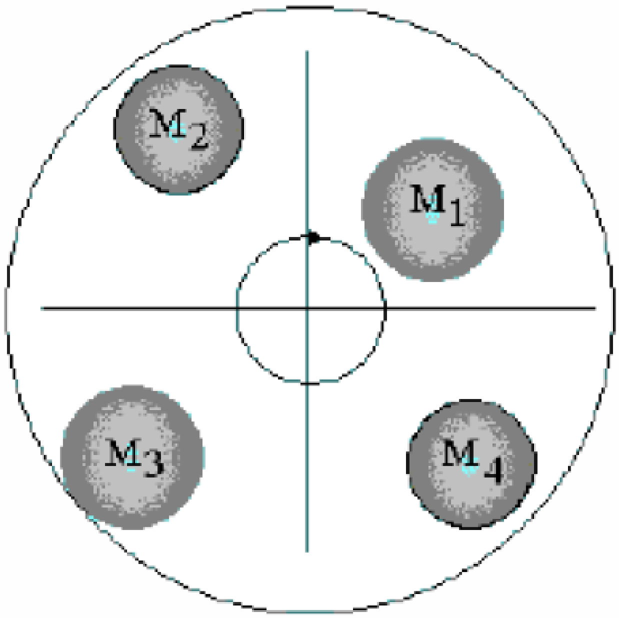
The radius of rotation is ***r*** and the angular velocity is ***ω*** in radians/second. The force acting on each one is hence:

$$F_1 = m_1 \cdot \omega^2 \cdot r_1 \;;\; F_2 = m_2 \cdot \omega^2 \cdot r_2;\; F_3 = m_3 \cdot \omega^2 \cdot r_3$$

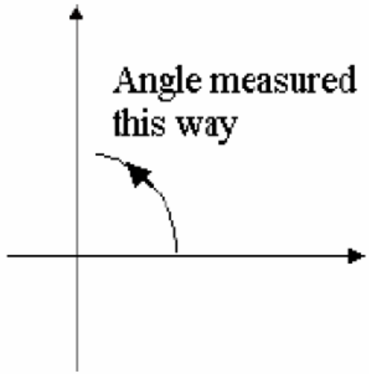
These are vector quantities and we can add them up to fine resultant for.



If the system was balanced, there would be no resultant force so the force needed to balance the system must be equal and opposite of the resultant (the vector that closes the polygon). The balancing mass M4 is then added at a suitable radius and angle such that the product m r is correct.



The result obtained would be the same whatever the value of ω and when ω = 0 we have static balance. In order to make the solution easier, we may make ω = 1 and calculate m r for each vector. This is called the m r polygon or vector Note that angles will be given in normal mathematical terms with anticlockwise begin positive from the x axis as shown.

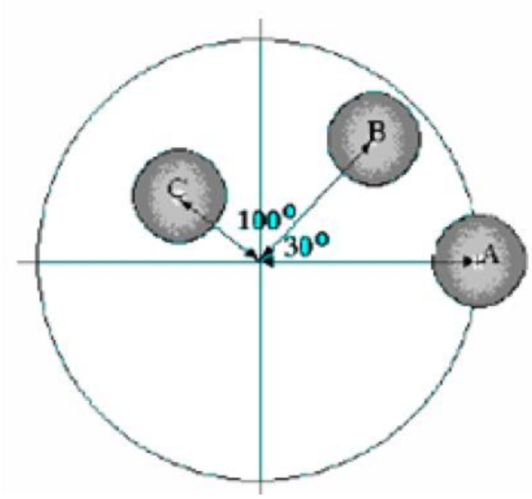


Example 1

Three masses A. B and C are placed on a balanced disc as shown at the figure below, radii of 120 mm. 100 mm and 80 mm respectively. The masses are 1 kg. 0.5 kg and 0.7 kg respectively. Find the 4th mass which should be added at a radius of 60 mm in order to statically balance the system **Analytical Solution and Graphically solution** .

1)Analytical Solution:

A more accurate approach to solving the vector diagrams in the preceding work is to resolve each vector into vertical and horizontal components. The resultant vector is then found by adding these components. Consider worked example above again.



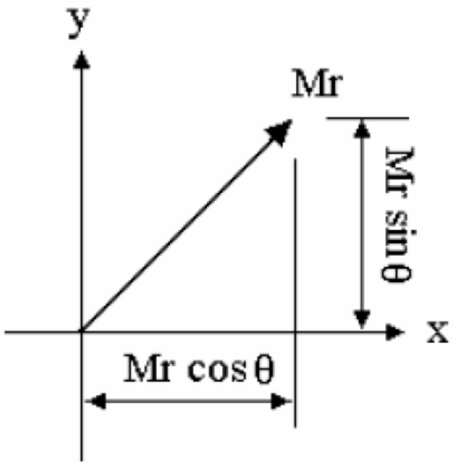
Each vector has a component in the x direction given by:

$$m \cdot r \cos\theta$$

and in the y direction given by:

$$m \cdot r\, sin\, \theta.$$

Work out these for each vector and include in the table.

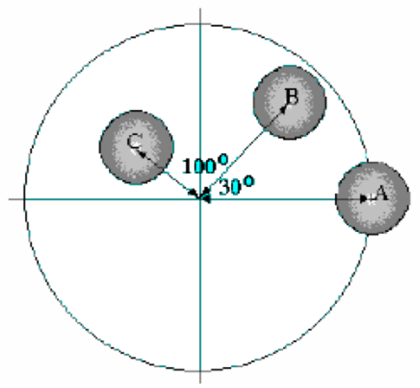


No.	Mass [Kg]	Radius [mm]	θ^0	Mr cos θ Kg mm	Mr sin θ Kg mm
A	1	120	0	120	0
B	0.5	100	30	43.3	25
C	0.7	80	130	-36	42.9
totals				127.3	67.9

The resultant vector has x and y component 127.3 Kg mm and 67.9 Kg mm respectively. This can be solved with Pythagoras. Resultant:

$$m \cdot r = \sqrt{127.3^2 + 67.9^2} = 144.3\; Kg \cdot mm\; as\; before$$

The mass required is 144/60=2.4 Kg.
The angle ϕ=tang^{-1}(67.9/127.3) =28^0
The balancing force is 180^0 anticlockwise of this so a balancing mass must be placed at angle of 208^0.



2)Graphically solution

First draw up a table to calculate the value of ***mr*** for each mass:

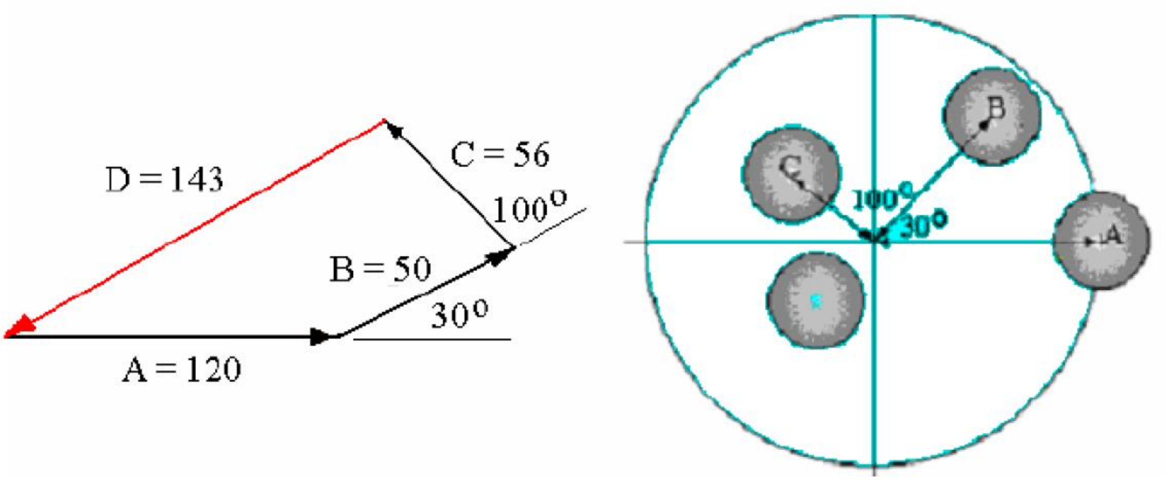
No.	Mass [Kg]	Radius [mm]	Mr [Kg mm]
A	1	120	120
B	0.5	100	50
C	0.7	80	56
D	m_D	60	60 m_D

Draw the ***mr*** polygon to fine the value of ***mr*** for 4th mass

The resultant 144.3 Kg m and is equal to 60 m_D. the mass required:

$$m_D \cdot r_D = 144.3 \xrightarrow{yields} m_D = \frac{144.3}{r_D} = \frac{144.3}{60} = 2.405\ Kg$$

$$204°\ anticlochwise\ of\ A\ as\ shown$$



Example 2

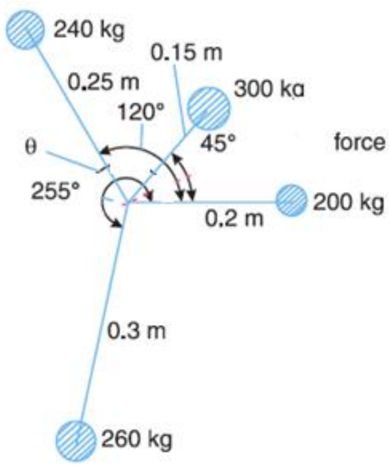
Four masses m1, m2, m3 and m4 are 200 kg, 300 kg, 240 kg and 260 kg respectively. The corresponding radii of rotation are 0.2 m, 0.15 m, 0.25 m and 0.3 m respectively and the angles between successive masses are 45°, 75° and 135°. Find the position and magnitude of the balance mass required **Graphically solution**, if its radius of rotation is 0.2 m.

Solution.

Given : $m_1 = 200$ kg ; $m_2 = 300$ kg ; $m_3 = 240$ kg ; $m_4 = 260$ kg ; $r_1 = 0.2$ m ; $r_2 = 0.15$ m ; $r_3 = 0.25$ m ; $r_4 = 0.3$ m ; $\theta_1 = 0°$; $\theta_2 = 45°$; $\theta_3 = 45° + 75° = 120°$; $\theta_4 = 45° + 75° + 135° = 255°$; $r = 0.2$ m.

Find the position and magnitude of the balance mass required, if its radius of rotation is 0.2 m.
Let m = Balancing mass, and θ = The angle which the balancing mass makes with m_1

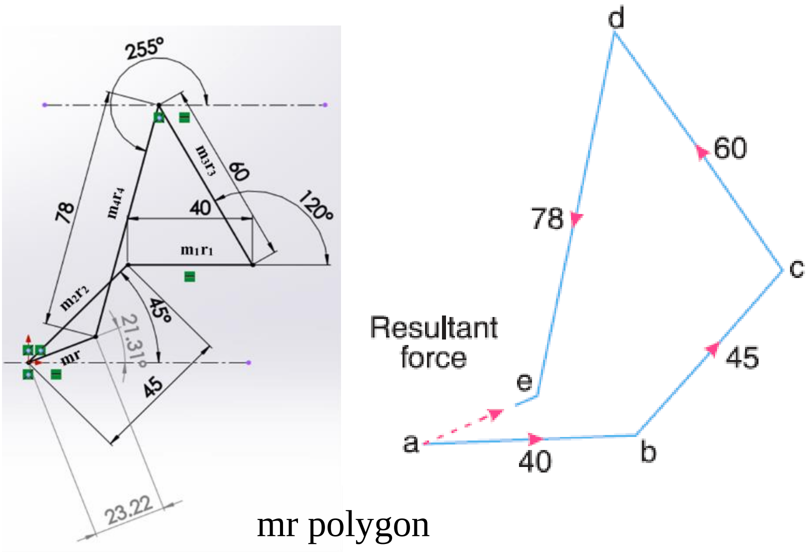
1- draw the space diagram showing the positions of all the given masses as shown in Figure below:



2- Tabulate the data as shown in Table below. The planes are tabulated in the same order in which they occur, reading from left to right.

No.	Mass [Kg]	Radius (r) [m]	mr [Kg m]
m_1	200	0.2	40
m_2	300	0.15	45
m_3	240	0.25	60
m_4	260	0.3	78
m	m	0.2	0.2m

3- Now draw the vector diagram with the above values, to some suitable scale, as shown in Figure below.



mr polygon

4- The closing side of the polygon *ae* represents the resultant force. By measurement, we find that *ae* = 23 kg-m.

5- The balancing force is equal to the resultant force, but ***opposite*** in direction as shown in figure of ***mr*** diagram. Since the balancing force is proportional to *m.r*, therefore:

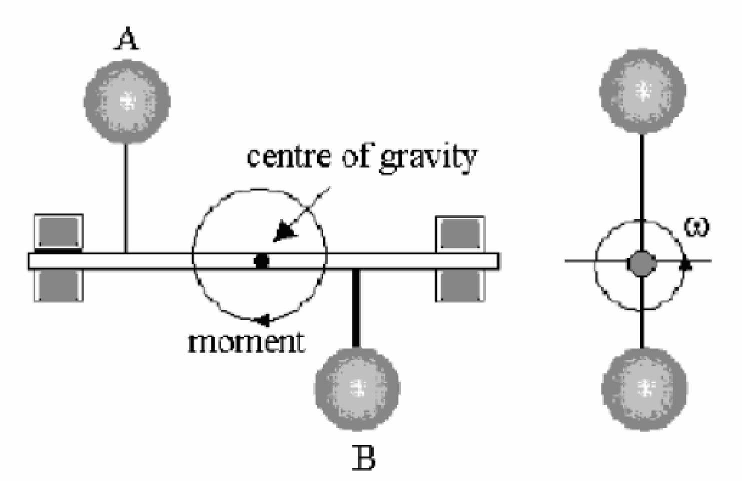
$$0.2 \cdot m = ae = 23Kgm \xrightarrow{yields} m = 115\ Kg$$

By measurement we also find that the angle of inclination of the balancing mass (*m*) from the horizontal mass of 200 kg:

$$\theta = 201°$$

4.4 Masses not in the same plane

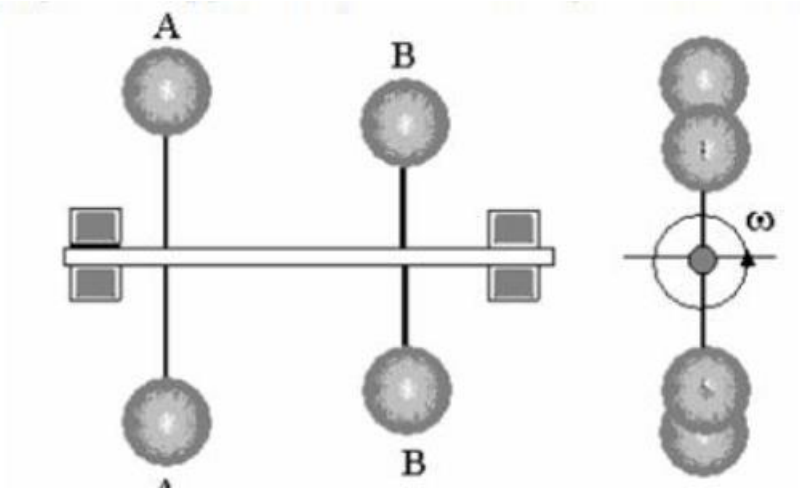
Consider 2 masses statically balanced as shown but acting at different places along the axis.



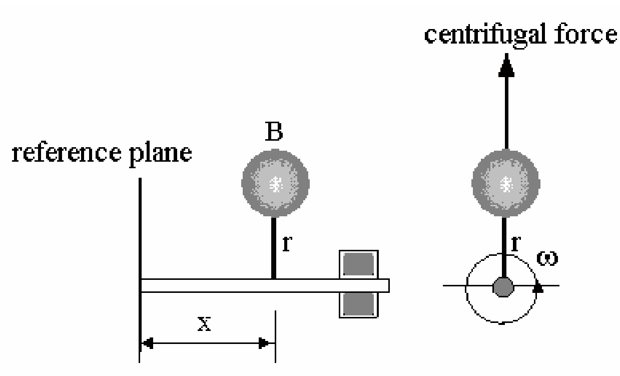
For static balance:

$$m_A r_A = m_B r_B$$

It is clear that even with static balance, centrifugal force will produce a turning moment about the center of gravity for the system. In this simple case, the problem is solved by adding equal and opposite forces at the two points as shown.



Consider the turning moment due to a single mass



The centrifugal force produced is:

$$F = m \cdot r \cdot \omega^2$$

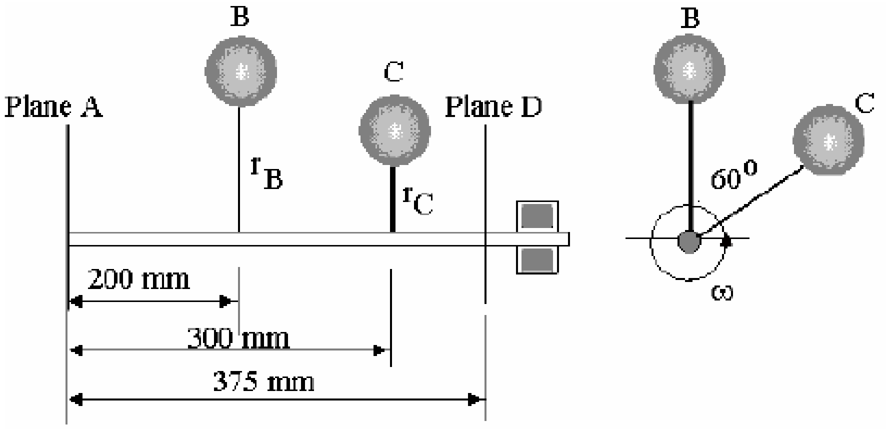
The turning moment about the reference plane:

$$T\,M = F \cdot x = m \cdot r \cdot \omega^2 \cdot x$$

For dynamic and static balance, we must work out the resultant turning moment and add masses at appropriate points to cancel it out. The appropriate points will be on two planes not coplanar with any of the original masses. This involves drawing two vector diagrams and since ω is common to all vectors we can a again take co =1 and draw vectors representing Mr and Mrx. Then calculate the required *masses* and angles.

Example3

Find the mass and the angle at which it should be positioned in planes A and D at radius of 60 mm in order to product complete balance of the system shown.



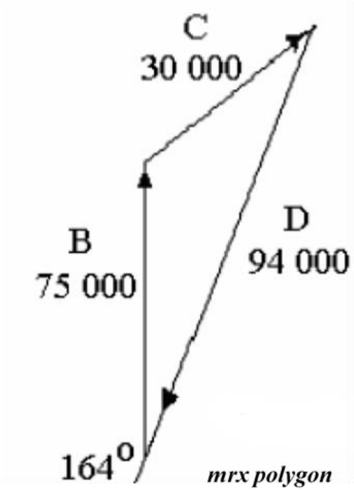
$\boldsymbol{r_B}$=75 mm; $\boldsymbol{r_C}$= 50 mm; $\boldsymbol{m_B}$=5 Kg; $\boldsymbol{m_C}$=2 Kg

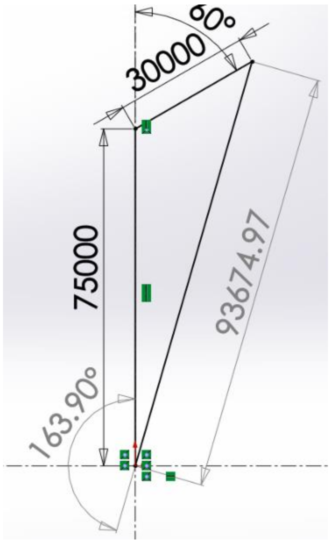
SOLUTION

Note that the diagram has been drawn with ***B*** vertical. It is a good idea to always start by making one of the known masses horizontal or vertical to make the construction of the vector diagrams easier. All angles should be expressed in absolute terms.

Plane A is the reference plane. All values of x are measured from plane A thus making ***mrx*** for A equal to zero. It follows that it does not appear in the vector diagram. Make up a table as follows leaving unknowns as symbols.

No.	m [Kg]	r [mm]	mr [Kg mm]	x [mm]	mrx [Kg mm^2]
A	m_A	60	60 m_A	0	0
B	5	75	375	200	75000
C	2	50	100	300	30000
D	m_D	60	60 m_D	375	22500 m_D





Now draw a polygon of ***mrx*** vectors in order to find the value of ***mrx*** at ***D***. Start with ***B*** in this case because it is vertical

Scaling the vector D which closes the tringle we find

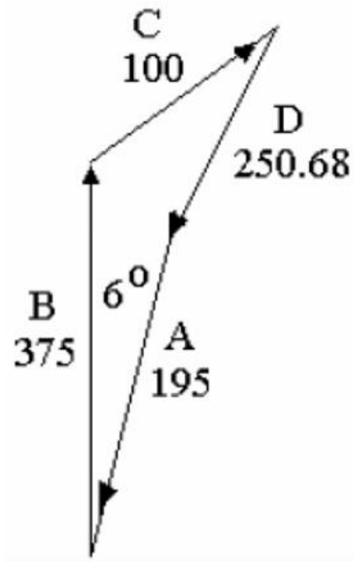
$$m \cdot r \cdot x\ for\ D = 22500\, m_D$$

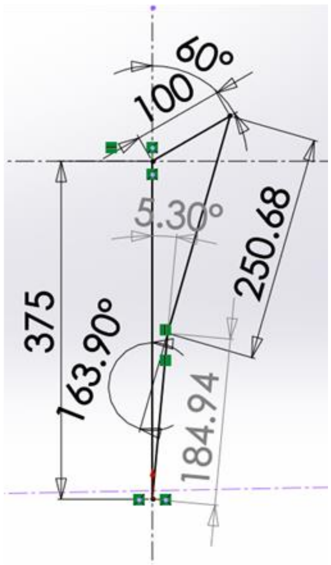
$mrx\ for\ D = 94000 = 22500 \cdot m_D$

$\xrightarrow{yields} m_D = \frac{94000}{22500} \approx 4 \cdot 17\ Kg; and\ it\ positioned\ 254°$

Now we calculate ***mr*** for ***D***:

$$mr\ for\ D = 4.17 \cdot 60 = 250.68$$



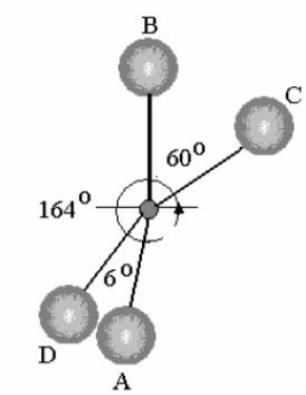


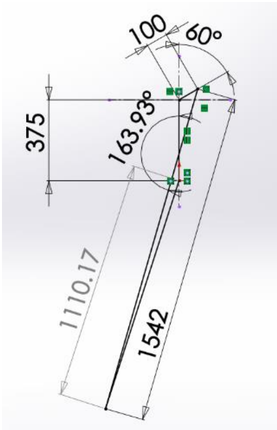
Next, we draw the polygon for the ***mr*** values as shown:

The vector which closed the polygon represents ***mr*** for A, ***mr*** for A=195Kgm

$$60\, m_A = 195Kgm \xrightarrow{yields} m_A = \frac{195}{60} = 3.25Kg\ at\ 6°\ to\ the\ vertical$$

The answer is best shown with an end view:





Example 4

A shaft carries four masses A, B, C and D of magnitude 200 kg, 300 kg, 400 kg and 200 kg respectively and revolving at radii 80 mm, 70 mm, 60 mm and 80 mm in planes measured from A at 300 mm, 400 mm and 700 mm. The angles between the cranks measured anticlockwise are A to B 45°, B to C 70° and C to D 120°. The balancing masses are to be placed in planes X and Y. The distance between the planes A and X is 100 mm, between X and Y is 400 mm and between Y and D is 200 mm. If the balancing masses revolve at a radius of 100 mm, find their magnitudes and angular positions.

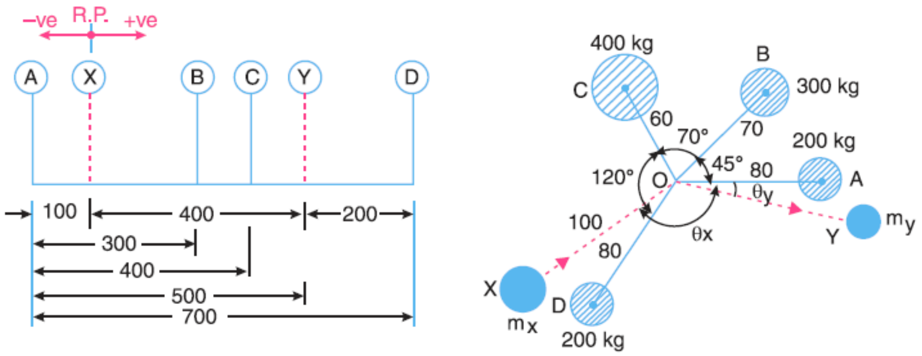
Solution.

Given: m_A = 200 kg ; m_B = 300 kg ; m_C = 400 kg ; m_D = 200 kg ; r_A = 80 mm = 0.08m ; r_B = 70 mm = 0.07 m ; r_C = 60 mm = 0.06 m ; r_D = 80 mm = 0.08 m ; r_X = r_Y = 100 mm = 0.1 m.

Let

m_X = Balancing mass placed in plane *X*, and m_Y = Balancing mass placed in plane *Y*.

The position of planes and angular position of the masses (assuming the mass A as horizontal) are shown in Figure (*a*) and (*b*) respectively.



All dimensions in mm.

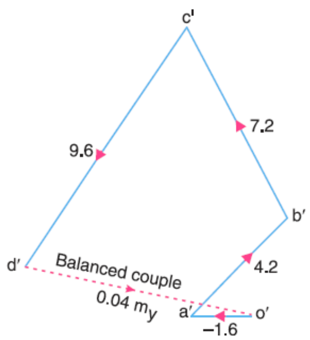
(*a*) Position of planes. (*b*) Angular position of masses.

Assume the plane *X* as the reference plane (*R.P.*). The distances of the planes to the right of plane *X* are taken as + ve while the distances of the planes to the left of plane *X* are taken as – ve. The data may be tabulated as shown in Table below:

Plane.	**m [Kg]**	**r [mm]**	**mr [Kg mm]**	**x [m]**	**mrx [Kg m^2]**
A	200	0.08	16	-0.1	-1.6
X (r.p)	m_X	0.1	0.1 m_X	0	0
B	300	0.07	21	0.2	4.2
C	400	0.06	24	0.3	7.2
Y	m_Y	0.1	0.1 m_Y	0.4	0.04 m_Y
D	200	0.08	16	0.6	9.6

The balancing masses m_X and m_Y and their angular positions may be determined graphically as discussed below:

1- draw the mrx diagram (couple polygon) from the data given in table above (column 6) as
shown in Figure (*c*) to some suitable scale:



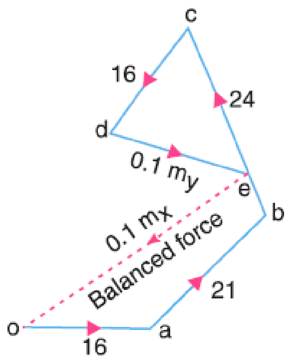
mrx **diagram**

2- The vector *d′o′* represents the balanced couple. Since the balanced couple is proportional to 0.04 m_Y, therefore by measurement:

$$0.04 \cdot m_Y = vector d'o' = 7.3\ Kgm^2 \xrightarrow{yields} m_Y = \frac{7.3\ Kgm^2}{0.04} = 182.5\ Kg$$

3- The angular position of the mass m_Y is obtained by drawing Om_Y in Figure (*b*), parallel to vector $d'o'$. By measurement, the angular position of m_Y is θ_Y = 12° in the clockwise direction from mass m_A (*i.e*. 200 kg).

4- Now draw the ***mr*** diagram (force polygon) from the data given in Table ***a*** (column 4) as shown in Figure(*d*).



mr diagram

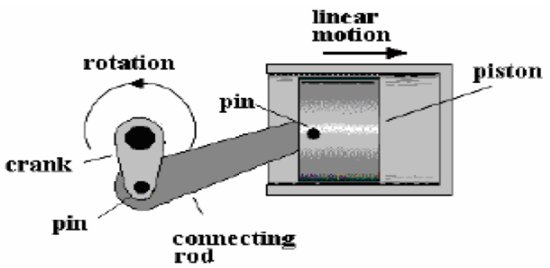
5- The vector ***eo*** represents the balanced force. Since the balanced force is proportional to 0.1 m_X, therefore by measurement:

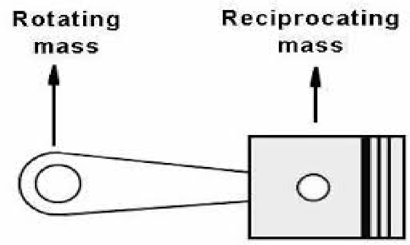
$$0.1 \cdot\ m_X = vector\ \boldsymbol{eo} = 35.5\ kgm \xrightarrow{yields} m_X = \frac{35.5\ kgm}{0.1m} = 355Kg$$

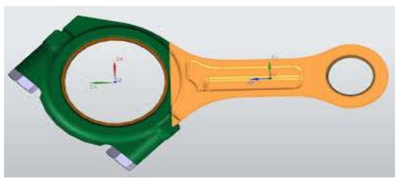
The angular position of the mass m_X is obtained by drawing Om_X in Figute. (*b*), parallel to vector ***eo***. By measurement, the angular position of m_X is θ_X = 145° in the clockwise direction from mass m_A (*i.e*. 200 kg).

4.5. Balance of Reciprocating Machines

Reciprocating machines here means a piston reciprocating in a cylinder and connected to a crank shaft by a connecting rod. You can skip the derivation of the acceleration by going to the next page-First let s establish the relationship between crank angle, and the displacement, velocity and acceleration of the piston.







4.6 Forces

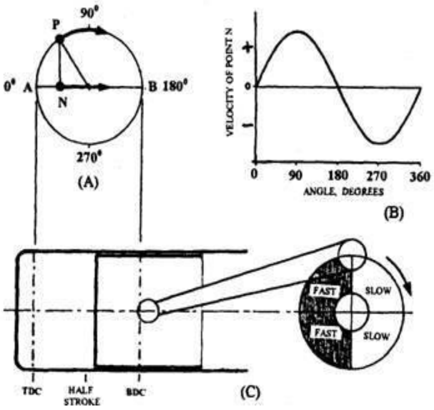
The internal combustion engines convert the linear motion of the pistons to the rotational motion of crankshaft. This attributes to the primary and secondary forces in an engine. A single cylinder four-stroke engine completes two rotations of crankshaft per power stroke and hence each cycle is given an angle of 180 degrees.

4.6.1 Primary forces- When a piston passes through TDC and BDC, the change of direction produces an inertia force due to which the piston tends to move in the direction in which it was moving before the change. This force, called the primary force, increases with the rise of the engine speed, and unless counteracted produces a severe oscillation in the vertical plane.

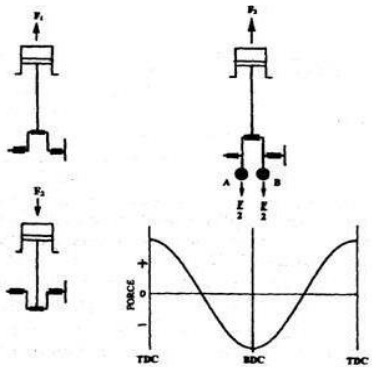
4.6.2 Secondary forces

In every cycle of 180 degree, the piston either moves from top to bottom or reverse. The distance travelled by the piston is not uniform and this give rise to the secondary forces. Secondary forces occur twice every half rotation and hence the name.

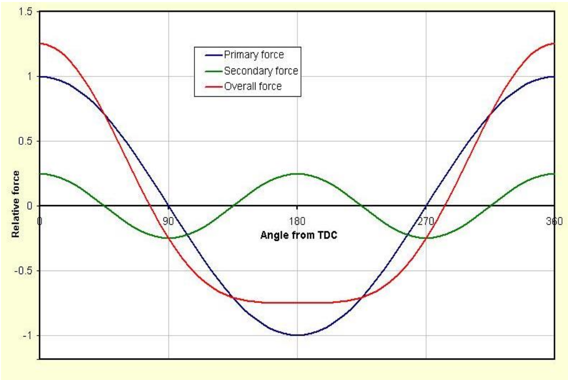
You could see that the distance moved by the piston by completing 90 degrees a little over the half stroke. Since piston is connected to the crankshaft and it rotates with uniform speed, the piston travels faster in first 90 degrees and slow in other half. The reverse happens while travelling from bottom to top.



Rise secondary force



Rise primary force



Reciprocation forces

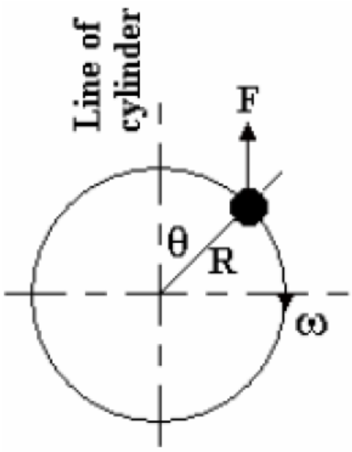
Using the close approximation fore acceleration, the inertial force required to accelerate the piston is given by:

$$a = \omega^2 \cdot R[cos\,\theta + \frac{\cos(2 \cdot \theta)}{n}]$$

This may be thought of as two separate forces:

$$F_p = m \cdot \omega^2 \cdot R \cdot cos\theta \; is\ called\ the\ primary\ forces$$

$$F_s = m \cdot \omega^2 \cdot R \left[\frac{cos(2 \cdot \theta)}{n}\right] is\ called\ the\ seondary\ force$$



4.7 Primary force for a single cylinder

The primary force:

$$F_p = m \cdot \omega^2 \cdot R \cdot cos\theta$$

Must be thought of as a force with peak value mω^{2}R that varies cosinusoidally with angle θ. m is mass of piston; R is crank radius. So, the primary force (F_p) is maximum when θ=0 or 180^0.

Example 5

Determine the primary out of balance force for a single cylinder machine with a piston of mass 0.5 kg, with a connecting rod 120 mm long and a crank radius of 50 mm when the speed of rotation is 3000 rev/mm.

Solution:

$$\omega = \frac{2 \cdot \pi \cdot n}{60} = \frac{2 \cdot \pi \cdot 300}{60} = 100 \cdot \pi$$

$$F_p = m \cdot \omega^2 \cdot R \cdot cos\theta = 0.5 \cdot (100 \cdot \pi)^2 \cos\theta = 2467.4 \cdot cos\theta \ N$$

4.8 Secondary force for a single cylinder

The secondary force

$$F_s = m \cdot \omega^2 \cdot R \left[\frac{cos(2 \cdot \theta)}{n}\right]$$

must be thought of as a force with peak value mω$_2$R/n what varies consensually with double angle 2θ. So, the primary force (F_s) is maximum when θ=0^0 ,90^0 180^0 and 360^0.

Example 6

Determine the secondary of balance force for a single cylinder machine with a piston of mass 0.5 kg, with a connecting rod 120 mm long and a crank radius of 50 mm when the speed of rotation is 3000 rev/mm.

Solution:

$$\omega = \frac{2 \cdot \pi \cdot n}{60} = \frac{2 \cdot \pi \cdot 300}{60} = 100 \cdot \pi; n = \frac{120}{50} = 2.4$$

$$F_s = m \cdot \omega^2 \cdot R \left[\frac{cos(2 \cdot \theta)}{n}\right] = 0.5 \cdot (100 \cdot \pi)^2 \cdot \frac{0.05}{2.4} \cdot cos(2 \cdot \theta)$$

$$= 1028.1 \cdot \cos(2 \cdot \theta)$$

Note: The unbalance forces due to reciprocating mass (piston) varies in magnitude but constant in direction, while due to rotating masses is constant in magnitude but varies in direction. 64

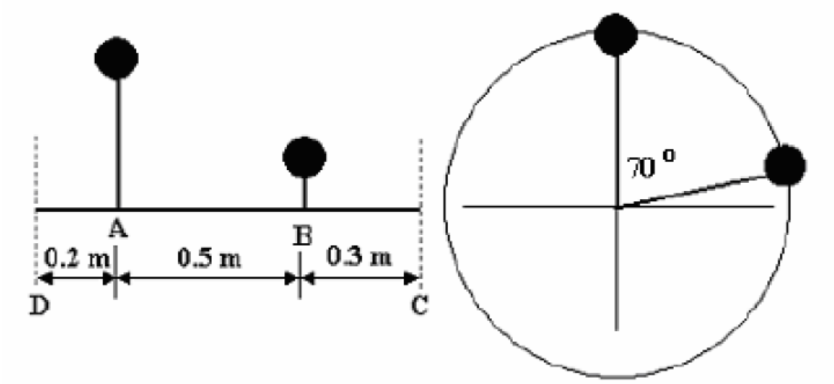
4.9 Problem in reciprocating Balance:

We know from the first balancing tutorial that in order to balance rotors we need to place balancing masses on two planes. Reciprocating machines can be balanced by placing two reciprocating masses on two planes. To balance primary components these would rotate at the crank speed. To balance secondary components, they would have to rotate at double the crank speed in order to produce double angles in a given period of time. The method so far used is easily adapted to solve the balance. We produce the **mrx** or **mrx/n** polygons and deduce the balancing component in the reference plane. Adding this component, we then draw the **mr** polygon to deduce the balancing component in the second reference plane.

4.9.1 Example 7

Two lines of reciprocating masses at A and B are to be balanced for PRIMARY forces and couples by two lines of reciprocating pistons at C and D. given m_A = 0.5 Kg and m_B=0.75Kg and that crank B is rotated 70^0 relatives to A, determine masses m_C and m_D and the angle of their cranks. All crank radii are the same.

Solution:



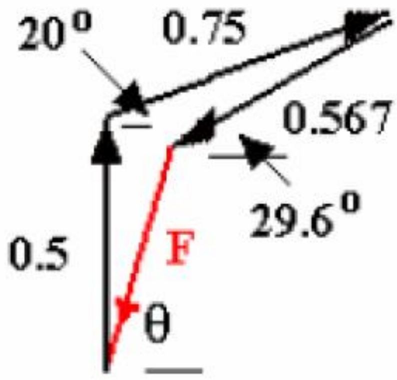
Space diagram

Make the D the reference plane

No.	Mass [Kg]	r [m]	x [m]	mr [Kgm]	Mrx [Kgm^2]
A	0.5	r	0.2	0.5r	0.1 r
B	0.75	r	0.7	0.75r	0.525 r
C	m_C	r	1	m_Cr	m_Cr
D	m_D	r	0	m_Dr	0

Draw the **mrx** polygon with a suitable scale ratio and with a bit of trigonometry:

m_Cr =0.567r; ϕ=29.6^0. the mass will be 0.567Kg placed on crank C at240.4^0 to crank A.

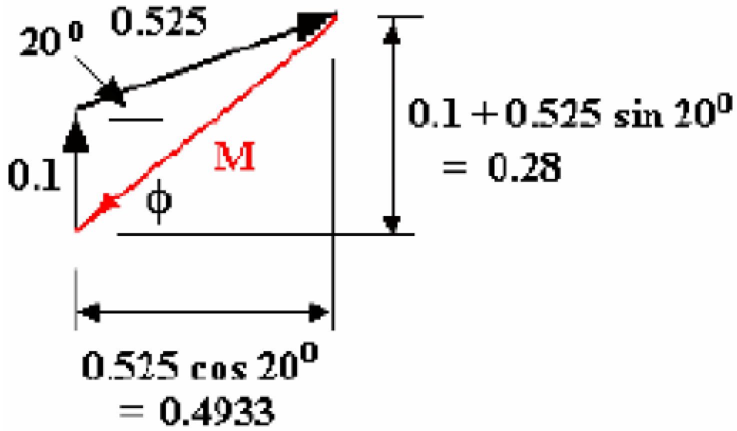


mr polygon

Now draw the **mr** polygon with:

$$m_C \cdot r = 0.567 \cdot 1, so\ m_C = 0.567$$

A bit more trigonometry or scaling from the diagram reveals that:
$F = 0.521 \cdot r; so\ m_D = 0.521Kg\ and\ it\ must\ be\ placed\ at\ 203.9°\ to\ rank\ A.$



mrx/n polygon