2.Velocity in Mechanisms:

In this chapter we shall discuss the velocity diagram and how it can be used to find such parameters.

2.1*Relative velocity of two bodies moving in straight line*.

Here we shall discuss the application of vectors for the relative velocity of two bodies moving along parallel lines and inclined lines, as shown in Fig. 2.1 *(a)* and *(b)* respectively.

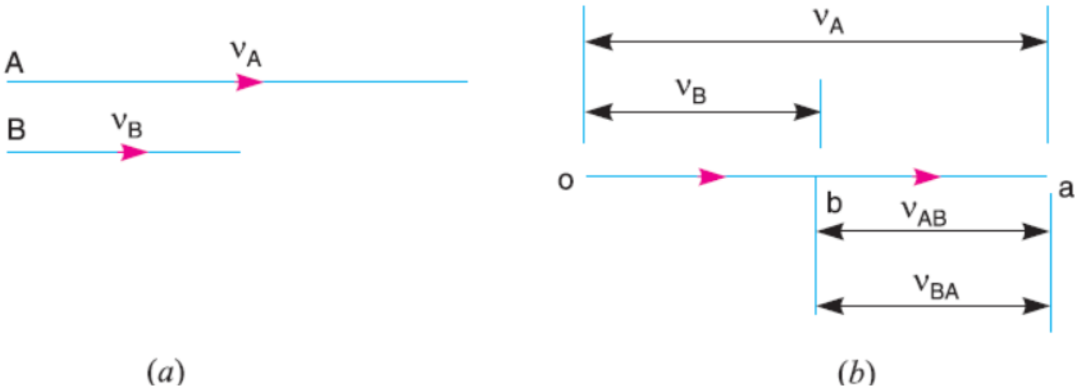


Fig 2.1. *Relative velocity of two bodies moving along parallel lines.*

Consider two bodies *A* and *B* moving along parallel lines in the same direction with absolute velocities v_A and v_B such that $v_A > v_B$, as shown in Fig.2.1 *(a)*. The relative velocity of *A* with respect to *B*:

$$v_{AB} = Vector\ diference\ of\ v_A\ and\ v_B = \overline{v_A} - \overline{v_B} \qquad (1)$$

From Fig. 2.1 (*b*), the relative velocity of *A* with respect to *B* (*i.e.* v_{AB}) may be written in the vector form as follows:

$$\overline{ba} = \overline{oa} - \overline{ob}$$

$$v_{AB} = -v_{BA}$$

Similarly, the relative velocity of *B* with respect to *A*,

$$v_{BA} = Vector\ diference\ of\ v_B\ and\ v_A = \overline{v_B} - \overline{v_A}$$

or

$$\overline{ab} = \overline{ob} - \overline{oa}$$

Now consider that the body B move in inclined direction as shown in fig 2.2 a, its relative velocity can be shown in fig 2.2b.

$$v_{AB} = Vector\ diference\ of\ v_A\ and\ v_B = \overline{v_A} - \overline{v_B}$$

or

$$\overline{ba} = \overline{oa} - \overline{ob}$$

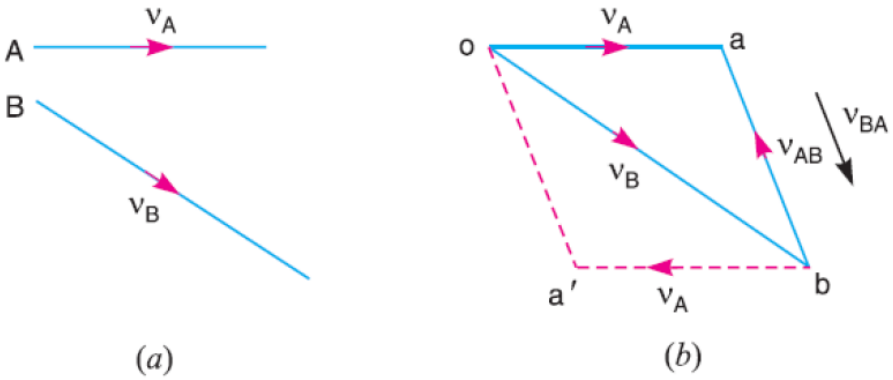


Fig 2.2. Relative velocity of two bodies moving along inclined lines.

Similarly, the relative velocity of *B* with respect to *A*,

$$v_{BA} = Vector\ diference\ of\ v_B\ and\ v_A = \overline{v_B} - \overline{v_A}$$

or

$$\overline{ab} = \overline{ob} - \overline{oa}$$

2.2 *Motion of a Link:*

Consider two points *A* and *B* on a rigid link *AB*, as shown in Fig. 2.3 (*a*). Let one of the extremities (*B*) of the link move relative to *A*, in a clockwise direction. Since the distance from *A* to *B* remains the same, therefore there can be no relative motion between *A* and *B*, along the line *AB*. It is thus obvious, that the relative motion of *B* with respect to *A* must be perpendicular to *AB*.

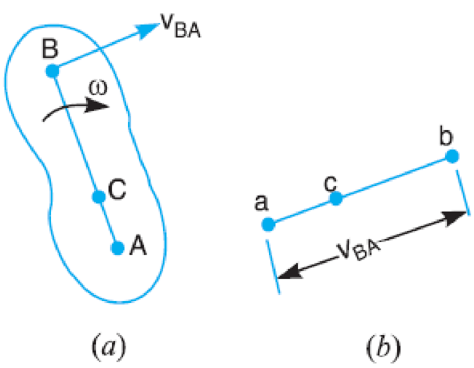


Fig 2.3 motion of a link

According to velocity method:

Velocity of any point on a rigid link with respect to another point on the same link is always perpendicular (⊥) to the line joining these points on the configuration (or space diagram).

The relative velocity of *B* with respect to *A* (*i.e.* v_{BA}) is represented by the vector *ab* and is perpendicular to the line *AB* as shown in Fig. 2.3 (*b*).

Let ω = Angular velocity of the link *AB* about *A*. We know that the velocity of the point *B* with respect to *A*,

$$v_{BA} = \overline{ab} = \omega \cdot AB \tag{1}$$

Similarly, the velocity of any point *C* on *AB* with respect to *A*,

$$v_{CA} = \overline{ac} = \omega \cdot AC \tag{2}$$

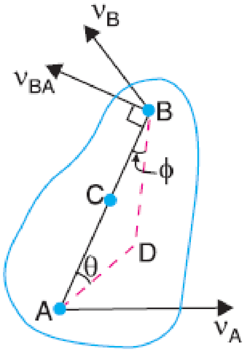
From equations **(1)** and **(2)**,

$$\frac{v_{CA}}{v_{BA}} = \frac{\overline{ac}}{\overline{ab}} = \frac{\omega \cdot AC}{\omega \cdot AB} = \frac{AC}{AB} \tag{3}$$

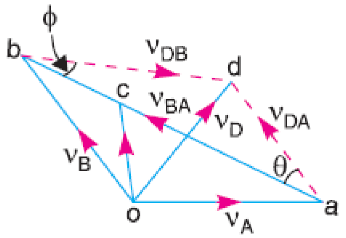
Thus, we see from equation (3), that the point ***C*** on the vector ***ab*** divides it in the same ratio as ***C*** divides the link ***AB***. Moreover, it is immaterial whether the link moves about ***A*** in a clockwise direction or about *B* in a clockwise direction.

2.3. Velocity of a Point on a Link by Relative Velocity Method

Consider two points A and B on a link as shown in Fig. 2.4 (a).



(*a*) Motion of points on a link.



(*b*) Velocity diagram.

Fig 2.4

Let the absolute velocity of the point ***A*** i.e. $\boldsymbol{v_A}$ is known in magnitude and direction and the absolute velocity of the point ***B*** i.e. $\boldsymbol{v_B}$ is known in direction only. Then the velocity of ***B*** may be determined by drawing the velocity diagram as shown in Fig. 2.4 (b). The velocity diagram is drawn as follows:

1. Take some convenient point *o*, known as the pole.
2. Through *o*, draw *oa* parallel and equal to v_A, to some suitable scale.
3. Through *a*, draw a line perpendicular to *AB* of Fig. 2.4 (*a*). This line will represent the velocity of *B* with respect to *A*, *i.e.* v_{BA}.
4. Through *o*, draw a line parallel to v_B intersecting the line of v_{BA} at *b*.
5. Measure *ob*, which gives the required velocity of point *B* (v_B), to the scale

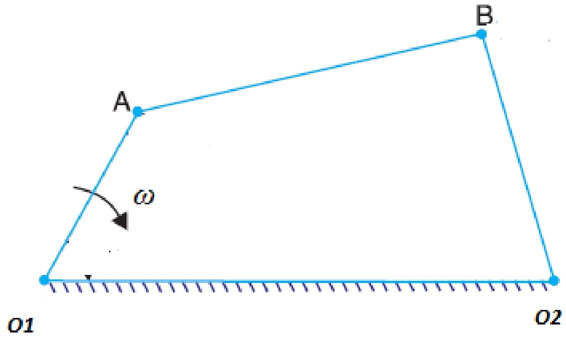
The absolute velocity of any point *C* on *AB* may be determined by dividing vector *ab* at *c* in the same ratio as *C* divides *AB* in Fig. 2.4 (*a*).In other words:

$$\frac{ac}{ab} = \frac{AC}{AB}$$

2.4. Application:

2.4.1. Four bar Mechanism:

1. Draw space diagram of mechanism with a suitable scale with higher case letter.



Space diagram

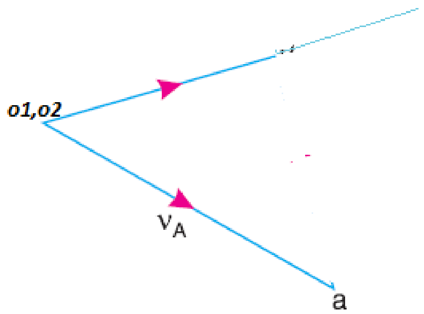
2. Taking a suitable scale to draw velocity diagram (with lower case letter) from

$$v_A = o_1A \cdot \omega$$

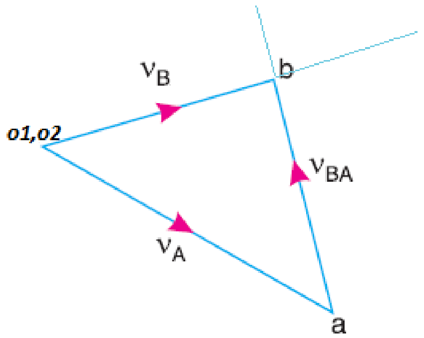
3.Take o_1, o_2 the origin which represent a fixed point from its absolute velocities are drawn.

*4.*Draw the known absolute velocity $v_A \perp O_{1A}$ from o_1.

5. The direction of absolute velocity v_B is known and can be drawn through o_2 where $v_B \perp O_2B$.



6. The relative velocity v_{AB} is drawn from (a) to be ⊥ to the link (AB) and intersect with v_B giving point (b). This establish the magnitudes of v_B, and v_{AB}.



Velosity diagram

2.2.2.1 Drawing scale

Is defined as A drawing that shows a real object with accurate sizes reduced or enlarged by a certain amount (called the scale).

2.2.2.2Three Types of Scale:

- Fractional or Ratio Scale: A fractional scale map shows the fraction of an object or land feature on the map. ...
- Linear Scale: A linear scale shows the distance between two or more prominent landmarks. ...
- Verbal Scale: This type of scale uses simple words to describe a prominent surface feature.

2.2.2.3 formula of Fractional or Ratio Scale:

$$S = \frac{DS}{AS}$$

Where is ***S*** is scale[-], ***DS*** is drawing scale [?], ***AS*** is actual scale [?.

Note:

1- (s) is Less than one when reduced (zoomed out) and greater than one when enlarged (zoomed in).

2- DS and AS must be of the same measurement unit.

Ex1:

Let us consider 4m length wall. Convert this to a scale of 1:50 and 1:100.

Solution: AS= 4m=4000mm, s=1:50 and 1:100, Ds=?

$$S = \frac{DS}{AS}$$

For s=1/50

$$\frac{1}{50} = \frac{DS}{4000} \xrightarrow{yields} DS = \frac{4000}{50} = 80mm$$

For s=1/100

$$\frac{1}{100} = \frac{DS}{4000} \xrightarrow{yields} DS = \frac{4000}{100} = 40mm$$

EX 2

In a four-bar chain **ABCD, AD** is fixed and is **150 mm** long. The crank **AB** is **40 mm** long and rotate at **120 rpm (cw)** while link **CD** is **80 mm** oscillate about **D. BC** and **AD** are of equal length. Find the angular velocity of link **CD** when angle **BAD** is **60º.**

Solution:

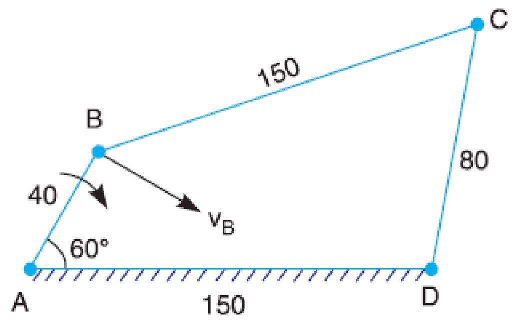
Solution. Given: N_{BA} = 120 r.p.m. or $\omega_{BA} = 2\,\pi \times 120/60 = 12.568$ rad/s

Since the length of crank *AB* = 40 mm = 0.04 m, therefore velocity of *B* with respect to *A* or velocity of *B*, (because *A* is a fixed point),

1) **Draw space diagram**

Find drawing scale of space diagram:

$$S = \frac{DS}{AS} = \frac{75mm}{150mm} = 1/2$$



(*a*) Space diagram (All dimensions in mm).

Space diagram with sacle 1/2

2) Find absolute velocity of crank

$$v_{BA} = v_B + v_A = v_B = \omega_{BA} \times AB = 12.568 \times 0.04 = 0.503\frac{m}{s}$$
$$= 50.3cm/sec$$

3) Draw velocity diagram

a) Find drawing scale of velocity diagram:

$$S = \frac{DS}{AS} = \frac{5.03cm/sec}{50.3cm/sec} = \frac{1}{10}$$

b). Since the link AD is fixed, therefore points ***a***, and ***d*** are taken as one part in the ***velocity diagram***.

c) Draw vector ***ab*** ⊥ ***AB*** (to ***scale*** in this case) the scale is

$$length\ factor\ ab = 50.3cm/sec \cdot \frac{1}{10} = 5.03cm/sec$$

3. From point ***b*** draw ***bc*** ⊥ ***CB***, to represent the velocity of *C* with respect to *B* (*i.e.* v_{CB})

4. From point ***d*** draw ***dc*** ⊥ ***CD***, represent the velocity of ***C*** with respect to ***D*** or simply velocity of ***C*** (*i.e.* $\mathbf{v_{CD}}$ or $\mathbf{v_C}$), so ***bc*** and ***dc*** will intersect at point ***c***.

5. By measuring ***dc*** is = 38.5 mm/sec

So, v_c=38.5x10=385 m*m*/*s*=0.385m/sec

And ω_{CD}=vc/CD=0.385/0.08= 4.8 rad/s

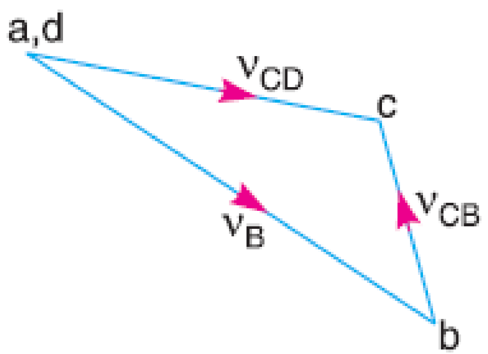


Fig. 2.6. Velocity diagram with scale drawing 1/10.

2.3. Slider Crank Mechanism

We have discussed the relative velocity method for the velocity of any point on a link, whose direction of motion and velocity of some other point on the same link is known. The same method may also be applied for the velocities in a slider crank mechanism.

2.3.1 Draw the space diagram

A slider crank mechanism is shown in Fig. 2.7. The slider ***A*** is attached to the connecting rod ***AB***. Let the radius of crank ***OB*** be ***r*** and let it rotates in a clockwise direction, about the point ***O*** with uniform angular velocity $\boldsymbol{\omega}$ rad/s. Therefore, the velocity of ***B*** i.e. $\boldsymbol{v_B}$ is known in magnitude and direction. The slider reciprocates along the line of stroke ***AO***.

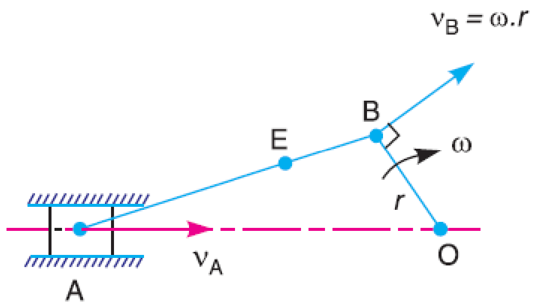
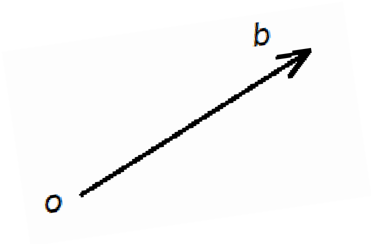


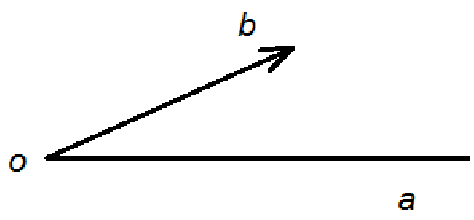
*Fig. 2.7 Slider crank mechanism. (***Space diagram)**

The velocity of the slider ***A*** (*i.e.* $\boldsymbol{v_A}$) may be determined by relative velocity method as discussed below:

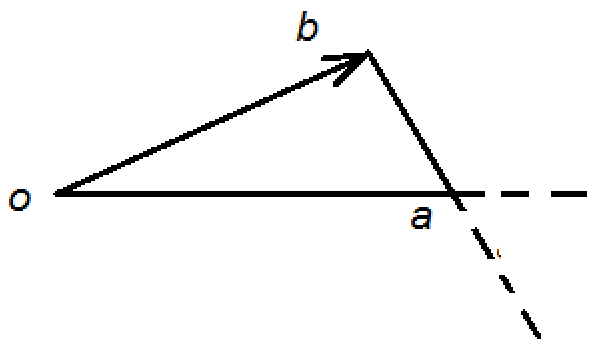
1- From any point ***o***, draw vector ***ob*** parallel to the direction of $\boldsymbol{v_B}$ (or perpendicular to ***OB***) such that $\boldsymbol{ob} = \boldsymbol{v}_{\boldsymbol{B}} = \boldsymbol{\omega}.\boldsymbol{r}$, to some suitable scale, as shown in below.



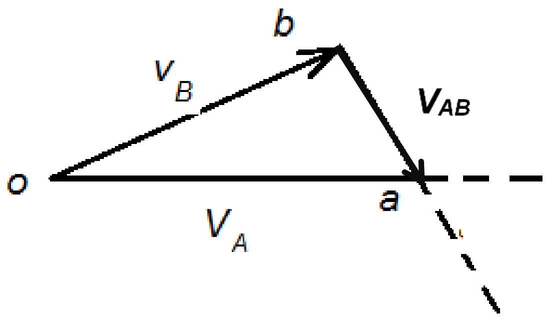
2 Draw vector ***oa*** parallel // to piston ***A*** known direction.



3. Since ***AB*** is a rigid link, therefore the velocity of ***A*** relative to ***B*** is perpendicular to ***AB***. Now draw vector ***ba*** perpendicular to ***AB*** to represent the velocity of ***A*** with respect to ***B*** i.e. v_{AB}.



4. The intersect of these two lines give point ***a***.
5. Then $\boldsymbol{v_A}$ can be measured



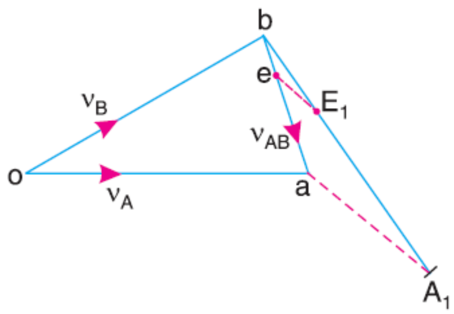
6. The angular velocity of the connecting rod ***AB*** ($\boldsymbol{\omega_{AB}}$) may be determined as follows:

$$\omega_{AB} = \frac{v_{BA}}{AB} = \frac{ab}{AB}\ (Anticlockwise\ about\ A)$$

Note: The absolute velocity of any other point E on the connecting rod ***AB*** may also be found out by dividing vector ***ba*** such that:

$$\frac{\overrightarrow{\boldsymbol{be}}}{\overrightarrow{\boldsymbol{ba}}} = \frac{\boldsymbol{BE}}{\boldsymbol{BA}}.$$

This is done by drawing any line $\boldsymbol{bA_1}$ equal in length of ***BA***. Mark $\boldsymbol{bE_1 = BE}$. Join ***a*** $\boldsymbol{A_1}$. From $\boldsymbol{E_1}$ draw a line $\boldsymbol{E_1e}$ parallel to $\boldsymbol{aA_1}$. The vector ***oe*** now represents the velocity of ***E*** and vector ***ae*** represents the velocity of ***E*** with respect to ***A***.



Ex. 3

The crank and connecting rod of a theoretical steam engine are 0.5 m and 2 m long respectively. The crank makes 180 rpm in the cw direction when it has turned 45° from the inner dead center position. determine the velocity of the piston and the angular velocity of the connecting rod.

Solution

Crank length=0.5=50cm, connecting rod length=2m=200cm

1) Draw space diagram

Find drawing scale of space diagram:

$$S = \frac{DS}{AS} = \frac{5cm}{200cm} = \frac{1}{20}$$

Space diagram with drawing scale 1/20

$$\omega_{OB} = \frac{2 \cdot \pi \cdot N}{60} = \frac{2 \cdot \pi \cdot 180}{60} = 18.852\ rad/s$$
$$v_B = \omega_{OB} \cdot \overline{OB} = 18.852 \cdot 0.5 = 9.426\ m/s$$

2) Draw velocity diagram

Find drawing scale of velocity diagram:

$$S = \frac{DS}{AS} = \frac{4.713\ m/sec}{9.426 m/sec} = \frac{1}{2}$$
$$v_B \xrightarrow{yields} \approx 4.75 \text{m/sec}$$

Follow the above procedure and when we measure the vector ***ob*** it gives 4.75 cm it means that

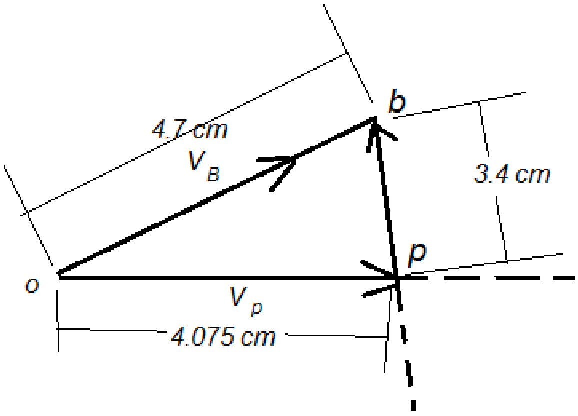
$$v_p = 2 \cdot 4.075 = 8.15\ m/s$$

and the vector ***bp*** gives 3.4 cm,

$$v_{BP} = 2 \cdot 3.4 = 6.8\ m/s$$

So

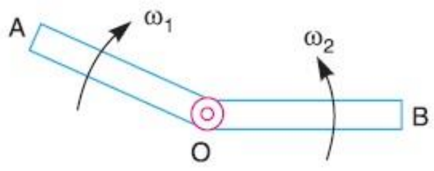
$$\omega_{PB} = \frac{v_{BP}}{BP} = \frac{6.8}{2} = 3.4\ rad/s$$



velocity polygon (diagram) with drawing scale1/2

2.4. Rubbing Velocity at a pin Joints

The links in mechanism are mostly connected by means of pin joints. The rubbing velocity is defined as "***The algebraic sum between the angular velocities of the two links which are connected by pin joint multiplied by the radius of the pin***". As shown below link OA , and OB connected by a pin joint O .



Links connected by pin joints.

Consider two links *OA* and *OB* connected by a pin joint at *O* as shown in Figure above:

$\boldsymbol{\omega_1}$ = Angular velocity of the link *OA* or the angular velocity of the point *A* with respect to *O*.

$\boldsymbol{\omega_2}$ = Angular velocity of the link *OB* or the angular velocity of the point *B* with respect to *O*, and

r = Radius of the pin.

According to the definition, Rubbing velocity at the pin joint *O*

$$v_{Rub} = (\omega_1 - \omega_2) \cdot r, if\ the\ links\ move\ in\ the\ same\ direction$$

$$v_{Rub} = (\omega_1 + \omega_2) \cdot r, if\ the\ links\ move\ in\ the\ opposite\ direction$$

Note: When the pin connects one sliding member and the other turning member then the angular velocity of the sliding member is Zero in such case $v_{Rub} = \omega r$.

Ex 4: (**Rubbing)**

For ***Ex 3*** find the rubbing velocities at the pins of the crank shaft, crank, and cross head when the pins diameters are 50 *mm*, 60 *mm*, and 30 *mm* respectively.

Solution:

We know that diameter of crank-shaft pin at *O*, d_0 = 50 *mm*=0.05 *m*

Diameter of crank-pin at *B*. d_B = 60 *mm*=0.06 *m*

and diameter of cross-head pin. d_c = 30 *mm*=0.03 *m*

We know that velocity of rubbing at the pin of crank-shaft

$$v_{RB} = \frac{d_\circ}{2} \cdot \omega_{BO} = \frac{0.05}{2} \cdot 18.85 = 0.47 m/s$$

Velocity of rubbing at the pin of crank

$$v_{RPB} = \frac{d_B}{2} \cdot (\omega_{BO} + \omega_{PB}) = \frac{0.06}{2} \cdot (18.85 + 3.4)$$

$$= 0.6675\ m/s\ (\omega_{BO} \text{ is clockwise and } \omega_{PB} \text{ is anticlockwise})$$

$$v_{RPB} = \frac{d_B}{2} \cdot (\omega_{BO} - \omega_{PB}) = \frac{0.06}{2} \cdot (18.85 - 3.4)$$

$$= 0.4635\ m/s\ (\omega_{BO} \text{ is clockwise and } \omega_{PB} \text{ is clockwise})$$

.

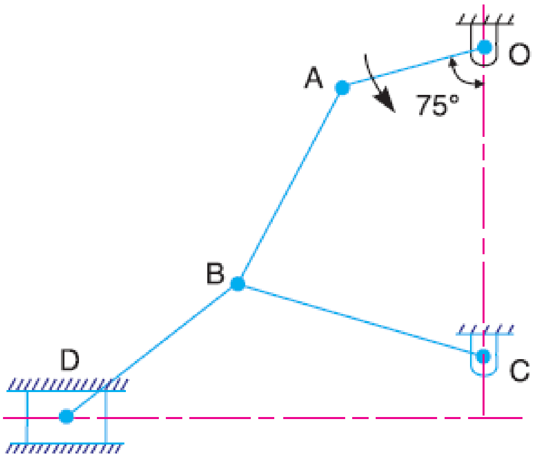
And velocity of rubbing at the pin of cross-head

$$v_{RA} = \frac{d_p}{2} \cdot \omega_{PB} = \frac{0.03}{2} \cdot 3.4 = 0.051 m/s$$

At the cross-head, the slider does not rotate and only the connecting rod has angular motion.)

Example 5

In figure below, the angular velocity of the crank OA is 600 r p.m. Determine the linear velocity of the slider D and the angular velocity of the link BD, when the crank is inclined at an angle of 75° to the vertical. The dimensions of various links are: OA = 28 mm; AB = 44 mm; BC 49 mm; and BD = 46 mm. The center distance between the centers of rotation O and C is 65 mm. The path of travel of the slider is 11 mm below the fixed point C. The slider moves along a horizontal path and OC is vertical.



Space diagram with scale1/20

Solution.

Given: N_{AO} = 600 r.p.m. or $\omega_{AO} = 2\pi \times 600/60 = 62.84$ rad/s, Since *OA* = 28 mm = 0.028 m, therefore velocity of ***A*** with respect to *O* or velocity of ***A*** (because *O* is a fixed point),

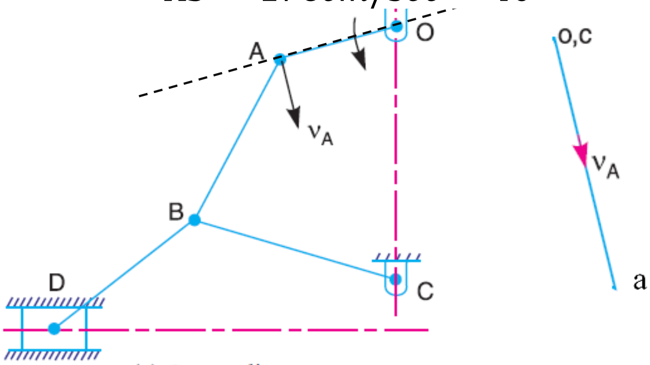
1) Draw space diagram with suitable scale 1/20

$$v_{AO} = v_A = \omega_{AO} \cdot OA = 62.84 \cdot 0.028 = 1.76 \frac{m}{s} = 176\ cm/sec,$$

$$\perp\ OA\ (Perpendicular\ to\ OA).$$

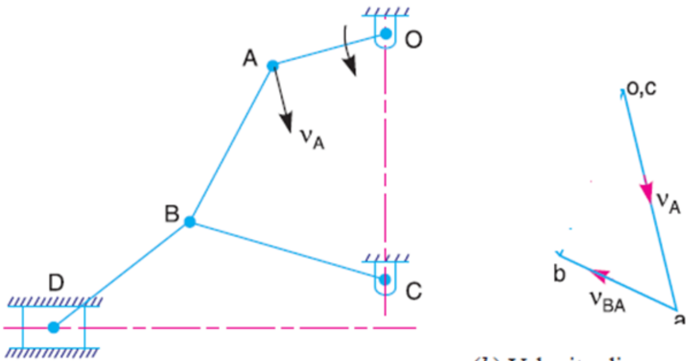
2) Draw vector *oa* = $v_{AO} = v_A$ = 1.76 m/s $\perp$ *OA* to some suitable scale.

$$S = \frac{DS}{AS} = \frac{4.4cm/sec}{176cm/sec} = \frac{1}{40}$$



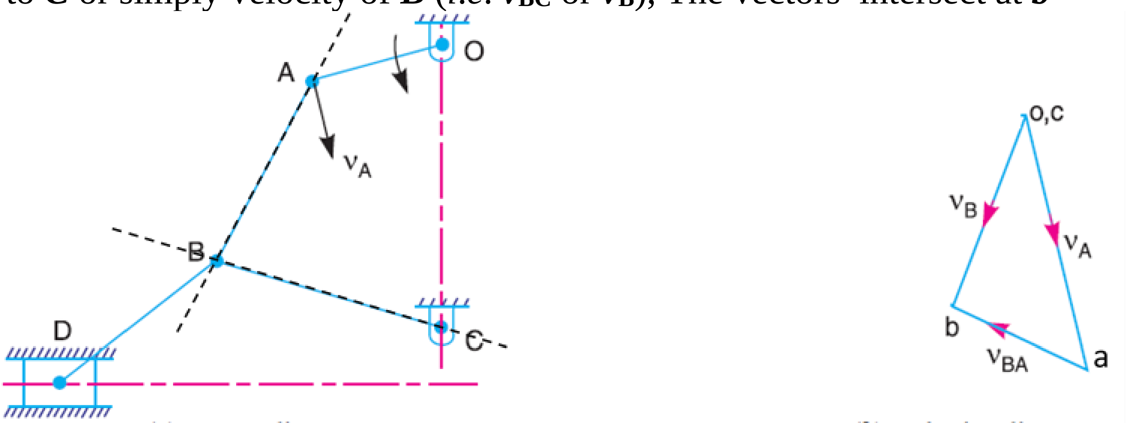
(*a*) Space diagram.

2- From point *a*, draw vector perpendicular to ***AB*** to represent the velocity of *B* with respect ***A*** (*i.e.* $\boldsymbol{v}_{\mathbf{BA}}$)



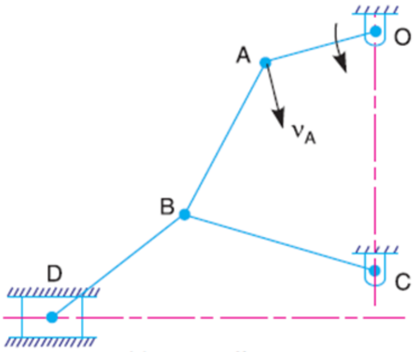
(*a*) Space diagram. (*b*) Velocity diagram.

3- From point ***c***, draw vector perpendicular to ***CB*** to represent the velocity of ***B*** with respect to ***C*** or simply velocity of ***B*** (*i.e.* $\boldsymbol{v}_{\mathbf{BC}}$ or $\boldsymbol{v}_{\mathbf{B}}$), The vectors intersect at ***b***

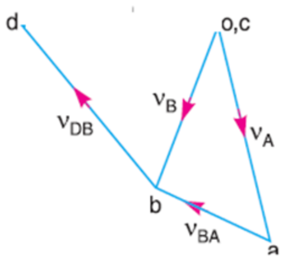


(*a*) Space diagram. (*b*) Velocity diagram.

4- From point ***b***, draw vector perpendicular to ***BD*** to represent the velocity of ***D*** with respect to ***B*** (*i.e.* $\boldsymbol{v}_{\mathbf{DB}}$).

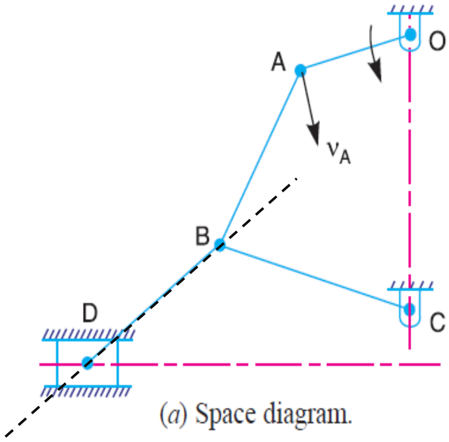


(a) Space diagram.

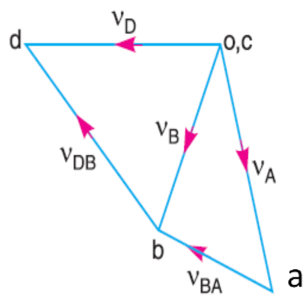


(b) Velocity diagram.

5- From point ***o***, draw vector parallel to the path of motion of the slider ***D*** which is horizontal, to represent the velocity of ***D*** (*i.e.* $\boldsymbol{v_D}$). The vectors intersect at ***d***.



(a) Space diagram.



(b) Velocity diagram.

6-By measurement, we find that velocity of the slider ***D***, $\boldsymbol{v_D}$ = vector *od* = 1.6 m/s By measurement from velocity diagram, we find that velocity of ***D*** with respect to ***B***, $\boldsymbol{v_{DB}}$ = vector ***bd*** = 1.7 m/s Since the length of link ***BD*** = 46 mm = 0.046 m, therefore angular velocity of the link ***BD***,

$$\omega_{BD} = \frac{v_{DB}}{BD} = \frac{1.7}{0.046} = 36.96\ rad/s\ \ (Clockwise\ about\ B)$$