Delivery Plan (Weekly Syllabus) المنهاج الاسبوعي النظري	
Week	**Material Covered**
Week 1	First Order Ordinary Differential Equations: Separable Equations
Week 2	First Order Ordinary Differential Equations: Linear Equations; Exact Equations
Week 3	Second Ordinary Differential Equations: Homogeneous; Non-Homogeneous
Week 4	Second Ordinary Differential Equations: The Euler Cauchy Differential Equations; Power Series Solutions
Week 5	Simultaneous Linear Differential Equations
Week 6	Simultaneous Linear Differential Equations
Week 7	Special Functions: Gamma Function
Week 8	Special Functions: Euler Beta Function
Week 9	Laplace Transform: - The General Method - The Transform of Special Functions
Week 10	Laplace Transform: - The Shifting Theorems - The Differentiation and Integration of Transforms - Solving Differential Equations by Laplace Transform
Week 11	Fourier Series - The Euler Formulas - Half Range Expansion
Week 12	Fourier Transform - Properties of Fourier Transform - Solving Differential Equations by Fourier Transform
Week 13	Orthogonality Properties of Sine and Cosine

Week 14	Partial Differential Equations -Separation of Variables (Heat Equations)
Week 15	Partial Differential Equations -Separation of Variables (Wave Equations)
Week 16	**Final Exam**

Learning and Teaching Resources مصادر التعلم والتدريس		
	Text	**Available in the Library?**
Required Texts	Advanced Engineering Analysis C. Ray Wylie.	Yes
Recommended Texts	Advanced Engineering Mathematics, 5th ed., D.G. Zill and M.R. Cullen.	Yes

<u>Differential Equation (D.E.)</u>

The differential equation is a relation between two variables. It consists of the two or more variables and the derivatives of one variable to others.

In general, there are two types of differential equations which are:

1- Ordinary differential equation (ODE)
2- Partial differential equation (PDE)

<u>Ordinary differential equation (ODE)</u>

An ordinary differential equation is an equation consists of an unknown function and its derivatives the unknown function depends on only one variable. The order of an ODE is the order of the highest derivative appearing in the equation.

For example:

$\frac{dy}{dx} = 3xy = e^x$ 1st-order ODE

$y'' + 4y' = 3y = 0$ 2st-order ODE

$y''' + 4y = sinx$ 3st-order ODE

An ODE can be classified according to the order of the equation into: -

1. First order ODE.
2. Second-order ODE.
3. High- order ODE.

1- <u>First order ODE</u>

A first order ODE has the following standard form: -

$$y' = f(x, y) \rightarrow \frac{dy}{dx} = \frac{M(x,y)}{N(x,y)}$$

$$and\ it\ can\ be\ written\ as\ follows:$$

$$M(x, y)dx \pm N(x, y)dy = 0$$

A First-order ODE can be classified into:

1- Separable Equation
2- Linear Equation
3- Exact Equation
4- Homogeneous Equation

1- Separable Equation

If a First-order ODE can be written in the form:

$$f(x)dx + g(y)dy = 0$$

It is called Separable Equation and the solution is:

$$\int f(x)\,dx + \int g(y)\,dy = c \quad \text{or} \quad \int g(y)\,dy = \int f(x)\,dx + c$$

Where:

f(x) is a function of x only

g(y) is a function of y only

c is a constant

Ex: solve $\frac{dy}{dx} = 2x\,(y^2+9)$ General Solution (G.S.) .

Sol: multiply by $\frac{dx}{y^2+9}$

$\Rightarrow \frac{1}{y^2+9}\,dy = 2x\,dx$

$\frac{1}{y^2+9}\,dy - 2\,x\,dx = 0\,,\quad \textit{the solution is}$

$\int \frac{1}{y^2+9}\,\text{dy} - \int 2x\,dx = c \Rightarrow \frac{1}{3}\tan^{-1}\left(\frac{y}{3}\right) - x^2 = c$

$\tan^{-1}\frac{y}{3} = 3c + 3\,x^2 \Rightarrow \frac{y}{3} = \tan\,(3c+3x^2)$

$y = 3\tan(3c + 3\,x^2) \Rightarrow y = \tan(k + 3\,x^2)\,, k = 3c$

Ex: solve $\frac{dy}{dx} = \frac{-x\cos x}{1-6y^5}$, $y(\pi) = 0$ Practical Solution (P.S.) like point (x,y)

Sol:

$(1-6y^5)dy = -x\cos x\,dx \Rightarrow (1-6y^5)dy + x\cos x\,dx = 0$

$\int (1-6y^5)dy + \int x\cos x\,dx = c$

$$\left\{ \begin{array}{c} u = x \Rightarrow \text{du} = \text{dx} \\ dv = \cos x\,dx \Rightarrow \text{v} = \sin x \end{array} \right\}$$

$\int x\cos x\,dx = x\sin x - \int \sin x\,dx$

$= x \sin x + \cos x$

$y - y^6 + x \sin x + \cos x = c$ $at\ x = \pi \Rightarrow y = 0$

$0 - 0 + \pi \sin \pi + \cos \pi = c \Rightarrow c = -1$

The complete solution is

$y - y^6 + x \sin x + \cos x + 1 = 0$

If the 1st order ODE has the form $y' = f(ax + by + c)$ it can be reduced to separable eq. as follows:

Let $u = ax + by + c \Rightarrow \frac{du}{dx} = a + b\frac{dy}{dx}$

Ex: solve $\frac{dy}{dx} = (4x - y + 1)^2$

Sol:

$u = 4x - y + 1 \Rightarrow \frac{du}{dx} = 4 - \frac{dy}{dx} \Rightarrow \frac{dy}{dx} = 4 - \frac{du}{dx}$

$\frac{dy}{dx} = (u)^2 \Rightarrow 4 - \frac{du}{dx} = u^2 \Rightarrow \frac{du}{dx} = 4 - u^2 \Rightarrow$ multiply by $\frac{dx}{4-u^2}$

$\frac{du}{4 - u^2} = dx \Rightarrow \frac{du}{4 - u^2}$ $sep. eq.$

$\int dx = \int \frac{du}{4-u^2} \Rightarrow x + c = \frac{1}{2} \tanh^{-1} \frac{u}{2}$

$x + c = \frac{1}{2} \tanh^{-1} \frac{4x - y + 1}{2}$

Ex: solve $x\frac{dy}{dx} = y + 4x^5 \cos^2(\frac{y}{x})$ ....* at $y(2) = 0$

Sol:

Let $u = \frac{y}{x} \Rightarrow \frac{du}{dx} = \frac{1}{x}\frac{dy}{dx} - \frac{y}{x^2}$

Eq. (*) $\div x^2 \Rightarrow \frac{1}{x}\frac{dy}{dx} - \frac{y}{x^2} = 4x^3 \cos^2(\frac{y}{x})$

$\left[\frac{du}{dx} = 4x^3 \cos^2 u\right] * \frac{dx}{\cos^2 u} \Rightarrow \frac{du}{\cos^2 u} = 4x^3\, dx$

$\int \sec^2 u\, du = \int 4x^3\, dx \Rightarrow \tan u = x^4 + c$

$\tan(\frac{y}{x}) = x^4 + c$

$at\ x = 2, y = 0$

$\Rightarrow tan\left(\frac{0}{2}\right) = 2^4 + c \Rightarrow c = -16$ The complete solution is:

$tan\left(\frac{y}{x}\right) = x^4 - 16$

Ex: solve $2y\frac{dy}{dx} + 3 = 0$

Sol:

$2y\frac{dy}{dx} + 3 = 0$ ……. multiply by dx

$2y\,dy + 3dx = 0$

$\int 2y\,dy + \int 3dx = 0$

$\frac{2\,y^2}{2} + 3x = c \Rightarrow y = \sqrt{c - 3x}$

Ex: solve $(1 + x^2)\,dy - (xy)dx = 0$

Sol:

$(1 + x^2)dy = (xy)dx$ ………….. multiply by $\frac{1}{y(1+x^2)}$

$\frac{dy}{y} = \frac{x\,dx}{1+x^2}$

$\int \frac{dy}{y} = \int \frac{x\,dx}{1+x^2} \Rightarrow ln\,(y) = \frac{1}{2}ln(1 + x^2) + c$

Ex: solve $\frac{dy}{dx} = \sqrt{xy}$

Sol:

$\frac{dy}{dx} = \sqrt{x}\,\sqrt{y}$

$\frac{dy}{\sqrt{y}} = \sqrt{x}\,dx \Rightarrow y^{\frac{-1}{2}}\,dy = x^{\frac{1}{2}}\,dx$

$\int y^{\frac{-1}{2}}\,dy = \int x^{\frac{1}{2}}\,dx$

$\Rightarrow 2y^{\frac{1}{2}} = \frac{2}{3}\,x^{\frac{3}{2}} + c$ $\Rightarrow y = \frac{1}{2}\sqrt{\frac{2}{3}\,x^{\frac{3}{2}} + c}$

Tutorial Sheet

1- $(x^2 + y\, x^{\,2})\, dy + (y^2 + y^2\, x^{\,2}) dx = 0$

2- $x\, y\, y' + \sqrt{1 + x^2 + y^2 + x^2\, y^{\,2}} = 0$

3- $3e^x \tan dx + (1 + e^x)\, sec^2 y\, dy = 0 \ldots\ldots .y(0) = \frac{\pi}{4}$

4- $\frac{dy}{dx} = \sin(x + y)$